Correcting noise in optical fibers via dynamic decoupling

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Outline

1. Introduction
2. CPMG dynamic decoupling
3. Our results
4. Conclusions
What is dynamic decoupling good for?

- Logical qubits
- Error correction techniques
- Error mitigation and quantum control
- Physical qubits
Previous work on using dynamic decoupling for photons

- Optical fibers:
  - Massar & Popescu
    - [NJP 9 158] polarization mode dispersion
    - 2007
  - Wu and Lidar
    - [PRA 70 062310] analytical proof
    - 2004

- Dephasing:
  - Lucamarini et al.
    - [PRA 83 032320]
    - [PRL 103 040502]
    - photons in a ring cavity
    - 2009/2011
Sending a photon through an optical fiber

We use a polarization maintaining fiber

changes in birefringence caused by changes in environment

~ 10m

Corrects for dephasing in two orthogonal directions

Only need to correct for dephasing in one direction
How our noise effects our states

Not all of the states of our qubits will be effected by the noise. We can see this by looking at the states on the Bloch sphere.

Our noise is modelled as being along the $Z$ axis

The $|0\rangle$ and $|1\rangle$ states are not effected by the noise.

However all other states will be effected, and this is something we need to correct for. The worst case scenario is the $|+\rangle$ and $|-\rangle$ states.
A simple example of dynamic decoupling using two $\pi$ pulses. At the end of our sequence the state is back in its original position on the Bloch Sphere.
CPMG dynamic decoupling

We want our dynamic decoupling sequence to be resistant to errors in the waveplates.

Carr-Purcell-Meiboom-Gill dynamic decoupling is resistant to errors in the waveplates for states $|+\rangle$ and $|−\rangle$ [Morton et al. PRA 71 012332 2005]
Modelling our fiber

The overall accumulated phase in one noise region is given by

$$\Delta \phi = \frac{2\pi}{\lambda} \Delta n \langle \Delta L \rangle \left[ \frac{\Delta L}{\langle \Delta L \rangle} \right]$$  \hspace{1cm} (1)
The $|+\rangle$ state gives the worst fidelity

\[ \alpha|0\rangle + \beta|1\rangle \rightarrow e^{i\theta} \alpha|0\rangle + e^{-i\theta} \beta|1\rangle \]

The fidelity of the state is given by

\[ F = \cos^2 \theta + (|\alpha|^2 - |\beta|^2)^2 \sin^2 \theta \]

This is smallest when $|\alpha|^2 = |\beta|^2$, and therefore this is the state with the largest error.

When $|\alpha|^2 = 0$ or $|\beta|^2 = 0$ then the fidelity is always 1.
How the number of waveplates effects fidelity

Here we assume $\langle \Delta L \rangle = 10m$, and $\sigma_{\Delta L} = 3m$ with a total fiber length of 10km.
How $\sigma_{\Delta L}$ and $\sigma_{\Delta \phi}$ effect fidelity

Here we assume 500 waveplates on a fiber of length 10km with $\langle L \rangle = 10m$. 
Conclusions

We have shown that using a CPMG sequence of dynamic decoupling we can correct for dephasing errors in an optical fiber.

When \( \langle \Delta L \rangle = 10m \), \( \sigma_{\Delta L} = 3m \) and \( \sigma_{\Delta \phi} = \pm 100 \) radians for a 10km fiber, 400 waveplates are required to achieve a fidelity suitable for BB84.

In future work we will go on and explore what happens if we introduce errors onto the waveplates.
Quantum Control For Generation of High-N00N States

N00N states - background

• Path entangled state \( \frac{|N0\rangle + |0N\rangle}{\sqrt{2}} \). Maximal path number entanglement.

• Applications:
  - beating the classical diffraction limit (lithography, microscopy…),
  - Heisenberg limited interferometry (gyroscopy, accelerometers, magnetometers…),
  - spectroscopy….

• Experimentally N00N states experiments with up to 6 photons:

• Problems of efficient generation and engineering with many photons. Conceptual issues.
Quantum Interferometric Lithography

Deposition rate:
\[ \Delta_N(\phi) = \left\langle \left( \hat{a}^{\dagger} + e^{-i\phi} \hat{b}^{\dagger} \right)^N \left( \hat{a} + e^{+i\phi} \hat{b} \right)^N \right\rangle \]

Classical input:
\[ \Delta_N(\phi) = \cos^{2N}(\phi / 2) \]

NOON input:
\[ \Delta_N(\phi) = \cos^{2}(N\phi / 2) \]

Super-resolution, beating the single-photon diffraction limit.

Boto, Kok, Abrams, Braunstein, Williams, and Dowling PRL 85, 2733 (2000)
Super-Resolution

\[ \frac{\lambda}{N} \]

\[ \lambda \]
Optical N00N states in modes $a$ and $b$ \( \frac{|N0\rangle + |0N\rangle}{\sqrt{2}} \), Unknown phase shift $\phi$ on mode $b$ so \( \hat{H} = b^\dagger b \).

Cramer-Rao bound $\Rightarrow \delta \phi \geq \frac{1}{N}$ “Heisenberg Limit!”.

Phase Estimation

- Phase shift measurement $\hat{P}_a = (-1)^{a^\dagger a}$
- $\Delta \hat{P} = \left| \frac{d\langle \hat{P} \rangle}{d\phi} \right| \delta \phi$
- $\delta \phi = \left| \frac{\sin(N\phi)}{N} \right|$ $\sim \frac{1}{N}$
Super-Sensitivity

\[ \Delta \varphi = \frac{\Delta \hat{P}}{\left| \frac{d\langle \hat{P} \rangle}{d\varphi} \right|} \]

\[ \Delta \varphi = 1/N = \text{Heisenberg Limit} \]

\[ \Delta \varphi = 1/\sqrt{N} = \text{shot-noise limit} \]
Many photon N00N state generator using a Kerr non-linearity

- Generate large N00N states using a cross-Kerr interaction and a source of photon number states.
  e.g. modify Christopher Gerry PRA (59) page 4095 (1999) macroscopic (single-mode) cat states.

\[ \hat{H} = \hbar K a^\dagger ab^\dagger b \]

- BUT required non-linearity very large.
N00N state generation by measurement-induced non-linearity

• Several schemes using:
  - linear optics,
  - single photon sources or OPO,
  - conditioned on photodetection measurements.


• But exponential resource scaling!

• Analogy to quantum computing - KLM scheme. Efficient quantum computation is possible using only beam splitters, phase shifters, single photon sources and photo-detectors.

N00N states and $\pm \pi/2$ relational Schrödinger cats

(a) A 50:50 beam splitter followed by a $\pi/2$ phase shift converts an input N00N state to a relational Schrödinger cat with components with relative phases $\pm \pi/2$.

\[
U_{ps1}(\pi/2) U_{bs}(50:50) \left\{ |N\rangle|0\rangle_F + |0\rangle|N\rangle_F \right\}
\propto \int_0^{2\pi} \frac{d\phi}{2\pi} e^{-iN\phi} \sqrt{\frac{N}{2}} e^{i\phi} \sqrt{\frac{N}{2}} e^{i(\phi-\pi/2)}
+ (-1)^N \int_0^{2\pi} \frac{d\phi}{2\pi} e^{-iN\phi} \sqrt{\frac{N}{2}} e^{i\phi} \sqrt{\frac{N}{2}} e^{i(\phi+\pi/2)}
\]

(b) achieves the reverse.
Generating relational Schrödinger cats by a simple interference procedure

• A simple experiment can demonstrate interference phenomena between two modes initially in a state with no prior phase correlations.

• Localisation of the relative phase plays a central role.
Localising relative optical phase

Representation in terms of coherent states,

\[ |\sqrt{n}e^{i\phi}\rangle_{\text{c.s.}} = e^{-n/2} \sum_{r=0}^{\infty} \sqrt{\frac{n^r}{r!}} e^{ir\phi} |r\rangle_F \]

The detection process acts on the input state as follows,

\[ |N\rangle |N\rangle_F \propto \int d\theta d\phi e^{-iN(\theta+\phi)} |\sqrt{N}e^{i\theta}\rangle |\sqrt{N}e^{i\phi}\rangle_{\text{c.s.}} : \]

\[ \rightarrow \int d\theta d\phi e^{-\frac{i}{2}(2N-r-l)(\theta+\phi)} c_{l,r}(\Delta) |\sqrt{N}e^{i\theta}\rangle |\sqrt{N}e^{i\phi}\rangle_{\text{c.s.}} \]

multi-valued:

\[ \pm \Delta_0 = \pm 2 \arccos \left( \sqrt{\frac{r}{r+l}} \right) \]

Gaussian approximation:

\[ \sim \exp \left[ -\left( \frac{1}{4} \right) (\Delta \pm \Delta_0)^2 \right] \]

\[ \Delta \equiv \phi - \theta. \]
Fidelities for converting relational Schrödinger cats to N00N states

- Generation of relational Schrödinger cat states is probabilistic.

- No range for $\pm \Delta_0$ picked out.

- Only $\pm \pi/2$ cats can be immediately converted to N00N states.

- Define a fidelity where $\Lambda_{N00N}$ maximises $F$.

$$F = \left| \frac{\langle 0|N| + e^{-i\Lambda_{N00N}}\langle N|0\rangle}{\sqrt{2}} \right|^2 \psi_{\text{converted}}$$

$$F \sim \cos^2 N \left( \frac{\Delta_0 - \pi/2}{2} \right)$$

whenever $\Delta_0 \sim \pi/2$.

Poor scaling whenever $\Delta_0$ different from $\frac{\pi}{2}$.
A beam splitter as a two mode rotation

• A beam splitter implements a rotation on two modes, angle \( \zeta \) determined by transmittivity.

• Acting on a state with perfectly defined relative phase:

(1) RELATIVE PHASE STRETCH

\[
\Delta_1 = \arctan \left( \frac{\tan(\Delta_0)}{\cos(2\zeta)} \right)
\]

\( \pm \Delta_0 \rightarrow \pm \pi/2 \)!!!

Desired \( \pm \pi/2 \) components are fixed points!

Breaks down by \( \Delta_0 = 0 \) (not a cat!)

(2) CHANGE OF MODE INTENSITIES
Stretching relational Schrödinger cat states

Stretching relational Schrödinger cat reduces overlap of cat components. A unitary transformation cannot be sufficient!

Simple measurement yields $|\psi_{r.s.c.}(\Delta_0)\rangle$ localised at $\pm \Delta_0$.

Additional beam splitter corrects relative phases but distorts the cat $U(50:50)|\psi_{r.s.c}(\Delta_0)\rangle$:

We want $|\psi_{r.s.c.}(\pi/2)\rangle$: Need to correct intensity difference!
Correction by feed-forward

\[\psi_{rsc}(\pm \Delta_0)\]

\[|0\rangle\]

Relative phase correction

Intensity correction

Variable beam splitter reflectivity \(\eta(l,r)\)

\[\eta = 2 - \frac{2}{[1+\cos(\Delta_0)]}\]

implemented by Mach-Zehnder setup
Summary – Scalable N00N state generation

- Analytic results for all aspects of scheme.

- Feedforward correction circuit is efficient!
  Photon loss probabilities \( \sim \) binomial in \( \frac{\text{total}}{\cos(\Delta_0)} \) photons

- Compatible with a twin beam source,
  \[ \rho_{\text{source}} = \sum p(n)|n\rangle\langle n| \]

- Relational schrödinger cat states produced by simple measurement procedure are done so efficiently.

Efficiency of scheme independent of size of required N00N state. Scaling to many photons, spending two thirds of the input photons on the localisation process suffices for \( F=0.99 \). Lose further fixed fraction to correction process.