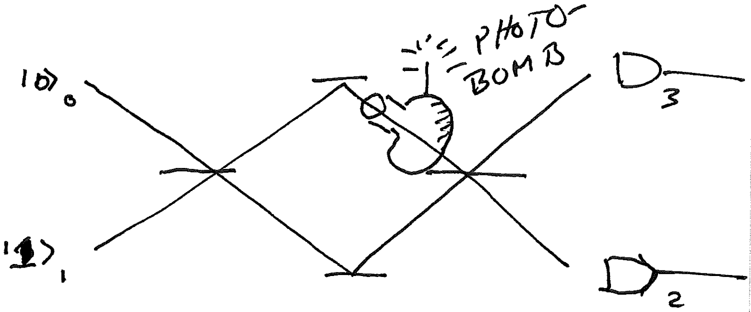
Consider the previous 1-photon interferometer with $\varphi = 0$, no phase shift.



This means paths are equal up to 2π modulo. Such an MZI is called "balanced".

$P_2(0) = 1 \qquad P_3(0) = 0$

Hence for a balanced MZI the photon always emerges from the lower port and is detected at D_2 (the bright port) while nothing comes out at D_3 (the dark port).

Let us spoil the interference by placing a bomb in the upper path, which has a hair-triggered photosensitive detonator. That is if a single photon hits the bomb — it goes off. We crudly modle the absorbing bomb by a nonunitary matrix $\hat{V} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ which deletes any photon in upper arm. Hence

$$\hat{T} = \hat{BS}\,\hat{V}\,\hat{BS}$$

$$= \frac{1}{2}\begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ i & 1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -1 & i \\ i & 1 \end{bmatrix}$$

where we have ignored $\hat{M} = \hat{\mathbb{1}}$ (overall phase) $\hat{\varphi} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ no phase shift.

~~$\Rightarrow \hat{a}_2 = (-\hat{a}_0 + i\hat{a}_1)/2 \qquad \hat{a}_3 = (i\hat{a}_0 + \hat{a}_1)/2$~~

Since $\det|\hat{T}| = 0$ there is no matrix inverse $\hat{T}^{-1}$

However we can extract an answer in the Schrödinger Picture via

$$|OUT\rangle = \hat{T}\,|IN\rangle$$

where $|IN\rangle = 0\,|1\rangle_0|0\rangle_1 + 1\,|0\rangle_0|1\rangle_1 = \left[\begin{matrix}0\\1\end{matrix}\right]^{0}_{1}$

$$\Rightarrow \quad |OUT\rangle = \frac{1}{2}\left[\begin{matrix}1 & i\\ i & 1\end{matrix}\right]\left[\begin{matrix}0\\1\end{matrix}\right] = \frac{1}{2}\left[\begin{matrix}i\\1\end{matrix}\right] = \frac{i}{2}|1\rangle_2|0\rangle_3 + 1\,|0\rangle_2|1\rangle_3$$

The state is not normalized as $\hat{T}$ is not unitary

Let $\hat{n}_2 = |1\rangle_2\langle 1|$ and $\hat{n}_3 = |1\rangle_3\langle 1|$

be the detector operators for D_2 and D_3

$$\Rightarrow \quad \bar{P}_2 = \langle OUT|\hat{n}_2|OUT\rangle = 1/4$$

$$\bar{P}_3 = \langle OUT|\hat{n}_3|OUT\rangle = 1/4$$

So now detector D_2 fires 25% of the time and D_3 fires 25% of the time. The missing 50% corresponds to photon absorbtion by the bomb [half the time photon takes bomb path].

Interpretation

25% D_2 clicks and we learn nothing

50% Bomb goes off

25% D_1 clicks and we learn bomb <u>must</u> be there but photon did not touch it