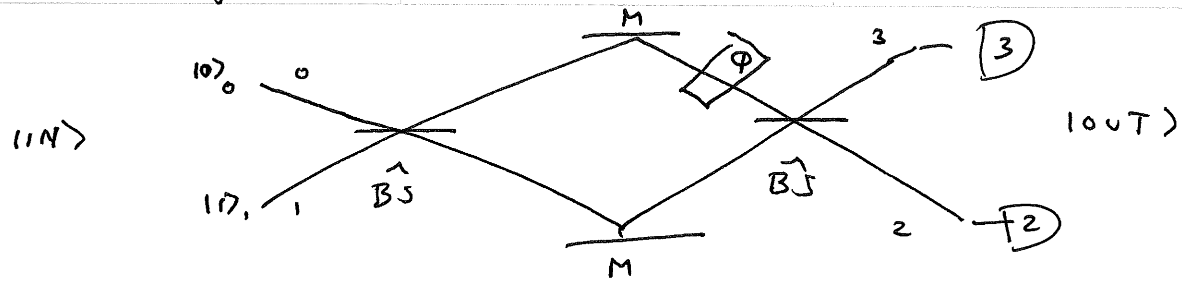
6.3 Single Photon Interferometer



Construct a single 2x2 <u>transfer</u> matrix $\hat{T}$ for MZI

Assume 50:50 BS

$$\hat{BS} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}$$

$$\hat{\varphi} = e^{i\hat{n}\varphi}$$

$$\hat{\varphi}\,|N\rangle = e^{iN\varphi}\,|N\rangle$$

$$\hat{\varphi}\,|\alpha\rangle = |e^{i\varphi}\alpha\rangle$$

$$\hat{M} = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$$ imparts $\pi/2$ phase shift per mirror

$$\hat{\varphi} = \begin{bmatrix} e^{i\varphi} & 0 \\ 0 & 1 \end{bmatrix}$$ imparts φ phase shift in upper branch.

$$|OUT\rangle = \hat{BS}\,\hat{\varphi}\,\hat{M}\,\hat{BS}\,|IN\rangle$$

$$= \hat{T}\,|IN\rangle$$

Hence $$\hat{T} = \frac{1}{2}\begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}\begin{bmatrix} e^{i\varphi} & 0 \\ 0 & 1 \end{bmatrix}\overbrace{\begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}}^{i\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}\begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}$$

mirror overall phase i can drop

$$= \frac{i}{2}\begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}\begin{bmatrix} e^{i\varphi} & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}$$

$$= \frac{i}{2}\begin{bmatrix} -1+e^{i\varphi} & i+ie^{i\varphi} \\ i(1+e^{i\varphi}) & 1-e^{i\varphi} \end{bmatrix}$$

It is easy to show $\hat{T}^{+}\hat{T} = \hat{1}$ is unitary

Dropping overall phase i out front

$$\hat{T} = \frac{1}{2}\begin{bmatrix} -(1-e^{i\varphi}) & i(1+e^{i\varphi}) \\ i(1+e^{i\varphi}) & 1-e^{i\varphi} \end{bmatrix}$$

Let $\hat{T} = \begin{bmatrix} t' & r \\ r' & t \end{bmatrix} \Rightarrow \quad r = \frac{i(1+e^{i\varphi})}{2} \qquad t = \frac{(1-e^{i\varphi})}{2}$

and $\quad t' = -\frac{(1-e^{i\varphi})}{2} \qquad r' = \frac{i(1+e^{i\varphi})}{2}$

which simpliy to

$$r = i e^{i\varphi/2}\cos(\varphi/2) \qquad r' = i e^{i\varphi/2}\cos\varphi/2$$

$$t = i e^{i\varphi/2}\sin(\varphi/2) \qquad t' = -i e^{i\varphi/2}\sin\varphi/2$$

$$\Rightarrow \quad |r|^2 = |r'|^2 = \cos^2\varphi/2 \qquad |r|^2+|t|^2 = \sin^2 + \cos^2 = 1$$

$$|t|^2 = |t'|^2 = \sin^2\varphi/2 \qquad |r'|^2+|t'|^2 = \sin^2+\cos^2 = 1$$

$$r^* t + r' t'^* = \tfrac{1}{2}\sin\varphi - \tfrac{1}{2}\sin\varphi = 0$$

$$r^* t' + r' t^* = -\tfrac{1}{2}\sin\varphi + \tfrac{1}{2}\sin\varphi = 0$$

$\hat{T}$ is a reciprocal device! We may treat like $\hat{BS}$!

$$\begin{bmatrix} \hat{a}_2 \\ \hat{a}_3 \end{bmatrix} = \hat{T}\begin{bmatrix} \hat{a}_0 \\ \hat{a}_1 \end{bmatrix} = \begin{matrix} t' a_0 + r\hat{a}_1 \\ r' a_0 + t\hat{a}_1 \end{matrix}$$

To invert $\det T = d = t't - r'r = e^{i\varphi}\sin^2\varphi/2 + e^{i\varphi}\cos^2\varphi/2$

$= e^{i\varphi}$

$$\Rightarrow \boxed{\hat{T}^{-1} = e^{-i\varphi}\begin{bmatrix} t & -r \\ -r' & t' \end{bmatrix}}$$

$$\Rightarrow \boxed{\begin{aligned} \hat{a}_0 &= e^{-i\varphi} t\,\hat{a}_2 + e^{-i\varphi}(-r)\,\hat{a}_3 \\ \hat{a}_1 &= e^{-i\varphi}(-r')\,\hat{a}_2 + e^{-i\varphi} t'\,\hat{a}_3 \end{aligned}}$$

$$\Rightarrow \quad \hat{a}_0^{+} = e^{i\varphi} t^*\,\hat{a}_2^{+} + e^{i\varphi}(-r^*)\,\hat{a}_3^{+}$$

$$\hat{a}_1^{+} = e^{i\varphi}(-r'^*)\,\hat{a}_2^{+} + e^{i\varphi} t'^*\,\hat{a}_3^{+}$$

$$\Rightarrow \quad |IN\rangle = |0\rangle_0 |1\rangle_1 \quad = \quad \hat{a}_1^{+} |0\rangle_0 |0\rangle_1$$

$$\overset{T}{\Rightarrow} \quad [\, e^{i\varphi} r^{*\prime} \hat{a}_2^{+} + e^{i\varphi} t^{\prime *} \hat{a}_3^{+} \,] \, |0\rangle_2 |0\rangle_3$$

$$= |OUT\rangle = \quad e^{i\varphi} r^{*\prime} |1\rangle_2 |0\rangle_3 + e^{i\varphi} t^{\prime *} |0\rangle_2 |0\rangle_3$$

$$= -e^{i\varphi} \left(i e^{i\varphi/2} \cos\varphi/2 \right)^* |1\rangle|0\rangle + e^{i\varphi} \left(-i e^{i\varphi/2} \sin\varphi/2 \right)^* |0\rangle|1\rangle$$

$$= -e^{i\varphi} (-i e^{-i\varphi/2} \cos\varphi/2) |1\rangle|0\rangle + e^{i\varphi} (+i e^{-i\varphi/2} \sin\varphi/2) |0\rangle|1\rangle$$

$$= \quad i e^{i\varphi/2} \cos\varphi/2 \, |1\rangle_2 |0\rangle_3 + i e^{i\varphi/2} \sin\varphi/2 \, |0\rangle_2 |1\rangle_3$$

$$\boxed{\begin{array}{llll} P_2 = \langle OUT | \hat{n}_2 | OUT \rangle = & \cos^2\varphi/2 & & \text{Detector 2} \\ P_3 = \langle OUT | \hat{n}_3 | OUT \rangle = & \sin^2\varphi/2 & & \text{Detector 3} \end{array}}$$

Hence choose $\varphi = 0$ (Balanced) and photons always/only arrive at D_2 (bright port) and never at D_3 (dark port). Choose $\varphi = \pi$ then MZI acts like a 50:50 B.S.

An MZI is like a BS with tunable t and r.

Note $P_2 + P_3 = 1$ so probability/energy conserved.