PHYS 4112 HW5

PROB 11.5.1.

$\Gamma(1/2) = \sqrt{\pi}$

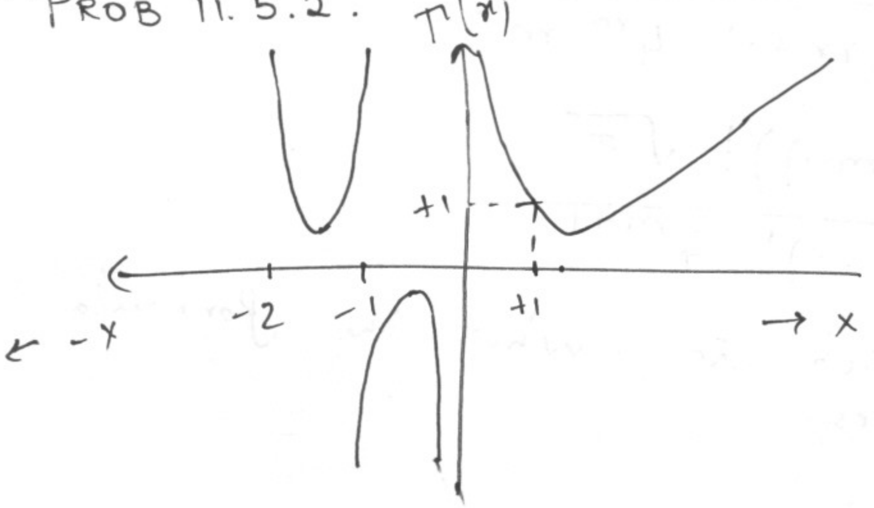
Using $\Gamma(p+1) = p\,\Gamma(p)$, $\Gamma(p) = \frac{1}{p}\Gamma(p+1)$

$\Gamma(3/2) = 1/2\,\Gamma(1/2) = \sqrt{\pi}/2$

$\Gamma(-1/2) = \frac{1}{-1/2}\Gamma(1/2) = -2\sqrt{\pi}$.

$\Gamma(-3/2) = \frac{1}{-3/2}\Gamma(-1/2) = -\frac{2}{3}\cdot(-2)\sqrt{\pi} = -\frac{4}{3}\sqrt{\pi}$.

PROB 11.5.2.



PROB 11.5.4.

P.T.

$$\Gamma(m+1/2) = \frac{1.3.5....(2m-1)\sqrt{\pi}}{2^n} = \frac{(2m)!\sqrt{\pi}}{4^m m!}$$

We know that $\Gamma(1/2) = \sqrt{\pi}$

Proof by induction

(1) Check 1^{st} term:

$$\Gamma(3/2) = \frac{\sqrt{\pi}}{2}$$

(2) Check $(n+1)^{th}$ term. ~~if~~ assuming it works for n^{th} term

$$\Gamma\left((m+1)+\tfrac{1}{2}\right) = \left(m+\tfrac{1}{2}\right)\Gamma\left(m+\tfrac{1}{2}\right)$$

$$= \left(m+\tfrac{1}{2}\right)\frac{(2m)!}{4^m\, m!}\sqrt{\pi} = \frac{(2m+1)}{2}\frac{(2m)!\sqrt{\pi}}{4^m\, m!}$$

$$= \frac{2m+1}{2}\cdot\frac{2m+2}{2m+2}\,\frac{(2m)!\ \sqrt{\pi}}{4^m\, m!}$$

$$= \frac{(2(m+1))!\ \sqrt{\pi}}{4(m+1)\ 4^m\ m!}$$

$$= \frac{(2(m+1))!\ \sqrt{\pi}}{(m+1)!\ 4^{m+1}}$$

$\equiv$ which is what the formula gives.

Hence proved.

ROB 11.7.6.

$$\int_0^\infty \frac{y\,dy}{(1+y^3)^2} = I$$

We use $\beta(p,q) = \int_0^\infty \frac{y^{p-1}}{(1+y)^{p+q}}\,dy$.

Substitute $y^3 = x \qquad 3y^2\,dy = dx\,, \quad y = x^{1/3}$.

$$I = \int \frac{x^{1/3}\cdot dx}{3\cdot x^{2/3}(1+x)^2} = \frac{1}{3}\int \frac{x^{-1/3}}{(1+x)^2}\,dx.$$

$$= \frac{1}{3}\,\beta\left(\frac{2}{3}, \frac{4}{3}\right).$$

$$= \frac{1}{3}\,\frac{\Gamma(2/3)\cdot\Gamma(4/3)}{\Gamma(2)}$$

$$= 0.403.$$

PROB. 11.7.7.

$$I = \int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}}$$

We use $\beta(p,q) = 2\int_0^{\pi/2} (\sin\theta)^{2p-1}(\cos\theta)^{2q-1}\,d\theta$

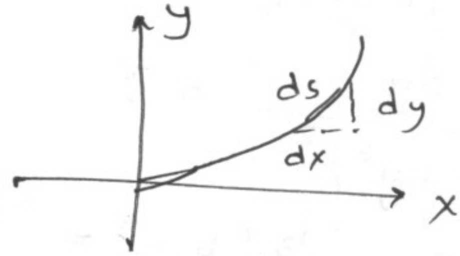
$$I = \int_0^{\pi/2} (\sin\theta)^{-1/2}(\cos\theta)^0\,d\theta$$

$$= \frac{1}{2}\,\beta\left(1/4, 1/2\right).$$

$$= 2.622$$

PROB 11.8.3.

$x = a(\theta + \sin\theta)$
$y = a(1 - \cos\theta)$.



$ds^2 = dx^2 + dy^2$.

$v = \sqrt{2gy}$

$= \dfrac{-ds}{dt}$. $\Rightarrow$ $t = \int \left|\dfrac{ds}{v}\right|$.

$dx = a\,d\theta\,(1+\cos\theta)$
$dy = a\sin\theta\,d\theta$.

$ds^2 = dx^2 + dy^2 = 2a. \dfrac{a^2\sin^2\theta\,d\theta^2}{a(1-\cos\theta)}$.

$= \dfrac{2a\ dy^2}{y}$

$\Rightarrow ds = \sqrt{\dfrac{2a}{y}}\,.\,dy$.

$v = \sqrt{2g(y_1 - y)}$ since it starts from y_1.

$t = \int \dfrac{ds}{v} = \int_0^{y_1} \sqrt{\dfrac{2a}{2gy(y_1-y)}}\,dy$.

$= \sqrt{\dfrac{a}{g}} \int \dfrac{dy}{\sqrt{y(y_1-y)}}$

Substitute $x = y/y_1 \Rightarrow dx = dy/y_1$, $x: 0 \to 1$.

$t = \int_0^1 \dfrac{y_1\,dx}{\sqrt{xy_1(y_1 - y_1x)}} = \int_0^1 \dfrac{dx}{\sqrt{x(1-x)}} = \beta\left(\frac{1}{2}, \frac{1}{2}\right) = \pi$