HW01HINTS
Assigned: THU 27 AUG 09
Due: THU 03 SEP 09

Problem 1. Boas Problem #1, Ch.10.2, page 501.

Problem 2. Boas Problem #3, Ch.10.2, page 501.
This can be done by recalling from class that the $\vec{\mathbf{A}}=\{a_{ij}\}$ matrix is a rotation from Alice to Bob and $\vec{\mathbf{A}}^T=\{a_{ji}\}$ is from Bob to Alice. Show the row and column vectors of $\vec{\mathbf{A}}$ are just the rotated unit vectors of Alice and Bob, so they must have the same properties.

Problem 3. Boas Problem #4, Ch.10.2, page 502.
Problem 2 above is the proof that $\sum_i a_{ij}a_{ik} = a_{ij}a_{ik} = \delta_{jk} = a_{ji}a_{ki} = \sum_i a_{ji}a_{ki}$ so you can use this to simplify the calculations.

Problem 4. Boas Problem #5, Ch.10.2, page 502.
Try using the Mathematica Solve command and then simplify using above hint.

Problem 5. Boas Problem #7, Ch.10.2, page 502.
Similar to proof in class using (a,b,c, ... z) "proof by induction" method.

Problem 6. In the hand out (also in the NRL Plasma Formulary), use the dyadic definition of a second rank tensor, Eq.(17), and its divergence, Eq.(18), to prove the tensor dyadic identities of Eqs. (19), (20), and (26). [For Eq.(26), note that I is the unit second rank tensor, which can be written as a 3-by-3 matix with ones on the diagonal and zeros off the diagonal.] Finally use the ordinary vector form of Gauss's Law, Eq.(28), to prove the tensor dyadic form of Gauss's Law, Eq.(29).

Note in Eq. (19) there is no dot so that $\vec{\mathbf{A}}\vec{\mathbf{B}} = \vec{\mathbf{T}} = \{A_iB_j\}$ is the *outer product* of the vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ and hence is a *second-rank tensor* whose components are $T_{ij} = A_iB_j$.
Treat

$$\vec{\nabla} = \begin{bmatrix} \partial_1 & \partial_2 & \partial_3 \end{bmatrix} = \begin{bmatrix} \partial/\partial x_1 & \partial/\partial x_2 & \partial/\partial x_3 \end{bmatrix} = \begin{bmatrix} \partial/\partial x & \partial/\partial y & \partial/\partial z \end{bmatrix}$$

as a vector *operator*. Hence $\vec{\nabla}\bullet\left(\vec{\mathbf{A}}\vec{\mathbf{B}}\right) = \vec{\nabla}\bullet\left(\vec{\mathbf{T}}\right)$ will be a *vector* as shown in class. Then as in class in Einstein summation notation we have for each component *j* of this vector:

$$\left[\vec{\nabla}\bullet\left(\vec{\mathbf{A}}\vec{\mathbf{B}}\right)\right]_j = \left[\vec{\nabla}\bullet\left(\vec{\mathbf{T}}\right)\right]_j = \partial_i T_{ij} = \partial_i\left(A_iB_j\right)$$

use the product rule for derivatives to split this into the sum of two terms and then compare term by term to the expression on the right, which is also a vector since $\vec{\nabla}\bullet\vec{\mathbf{A}} = \partial_i A_i$ is a scalar multiplying each component of $\vec{\mathbf{B}}$ and $\vec{\mathbf{A}}\bullet\vec{\nabla} = A_i\partial_i$ is a scalar operator that is operating on each component of $\vec{\mathbf{B}}$.

In Eq. (20) f is a scalar function of **r**. Hence $\vec{\nabla} \bullet \left(f\overleftrightarrow{\mathbf{T}} \right)$ is a vector with j components

$$\left[\vec{\nabla} \bullet \left(f\overleftrightarrow{\mathbf{T}} \right) \right]_j = \partial_i \left(f T_{ij} \right)$$

use the product rule for derivatives again to split this into the sum of two terms and then show component by component that this is equal to the two things on the right — both of which are also vectors. Remember from class that dotting a vector into a 2nd-rank tensor gives back a vector, and note $\vec{\nabla} f$ is a vector and $\vec{\nabla}$ is a vector operator so that $\vec{\nabla} \bullet \overleftrightarrow{\mathbf{T}}$ is a vector.

Eq.(26) There is no dot! This is the *outer product* of the vector operator $\vec{\nabla} = \left[\begin{array}{ccc} \partial / \partial x & \partial / \partial y & \partial / \partial z \end{array} \right]$ with the vector $\vec{\mathbf{r}} = \left[\begin{array}{ccc} x & y & z \end{array} \right]$ and hence the result is a rank-2 tensor. Write this out as a 3 by 3 matrix and show the diagonal is all ones and the off diagonals all zeros.