Eq. 15.1

$$\boxed{\frac{d}{dx}\left[x^{p} J_{p}\right] = x^{p} J_{p-1}}$$

Proof $p = m = 0, 1, 2, \ldots$

$$x^{p} J_{p}(x) = x^{p} \sum_{n=0}^{\infty} \frac{(-1)^n}{\Gamma(n+1)\Gamma(n+1+p)} \left(\frac{x}{2}\right)^{2n+p}$$

$$= x^{m} J_{m}(x) = x^{m} \sum_{n=0}^{\infty} \frac{(-1)^n}{n!\,(n+m)!} \left(\frac{x}{2}\right)^{2n+m}$$

$$= \left(2\,\frac{x}{2}\right)^{m} \sum_{n=0}^{\infty} \frac{(-1)^n}{n!\,(n+m)!} \left(\frac{x}{2}\right)^{2n+m}$$

$$= 2^{m} \sum_{n=0}^{\infty} \frac{(-1)^n}{n!\,(n+m)!} \left(\frac{x}{2}\right)^{2n+2m}$$

$$\Rightarrow \frac{d}{dx}\left[x^{m} J_{m}\right] = 2^{m} \sum_{n=0}^{\infty} \frac{(-1)^n\,\cancel{2}(n+m)}{n!\,(n+m)!} \left(\frac{x}{2}\right)^{2n+2m-1} \cdot \frac{1}{\cancel{2}}$$

$$= 2^{m} \sum_{n=0}^{\infty} \frac{(-1)^n}{n!\,(n+m-1)!} \left(\frac{x}{2}\right)^{2n+2m-1}$$

$$= \cancel{2^{m}} \left(\frac{x}{\cancel{2}}\right)^{m} \sum_{n=0}^{\infty} \frac{(-1)^n}{n!\,(n+m-1)!} \left(\frac{x}{2}\right)^{2n+(m-1)}$$

$$= \underbrace{x^{m} \sum_{n=0}^{\infty} \frac{(-1)^n}{n!\,[n+(m-1)]!} \left(\frac{x}{2}\right)^{2n+(m-1)}}_{}$$

$$x^{m} J_{m-1}(x) \quad \boxed{\therefore}$$