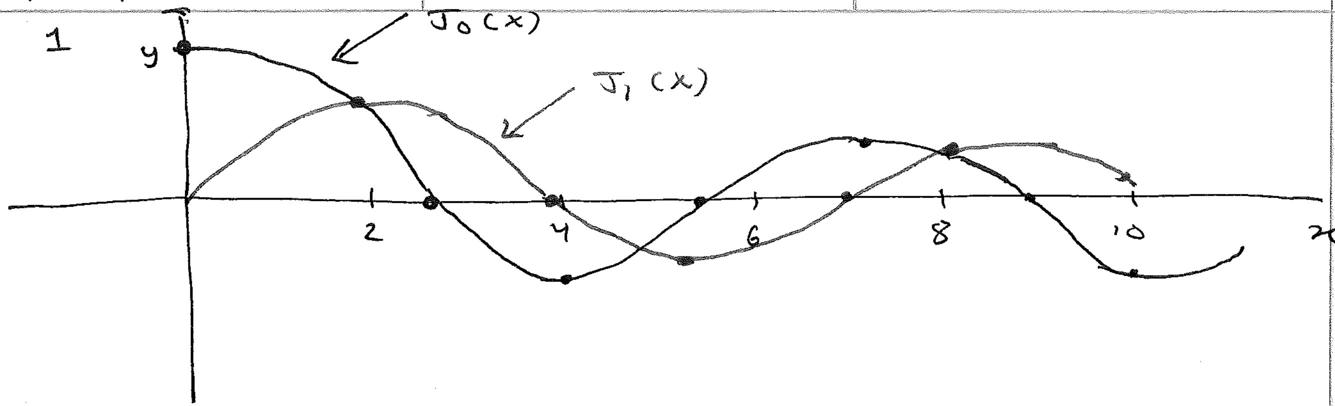

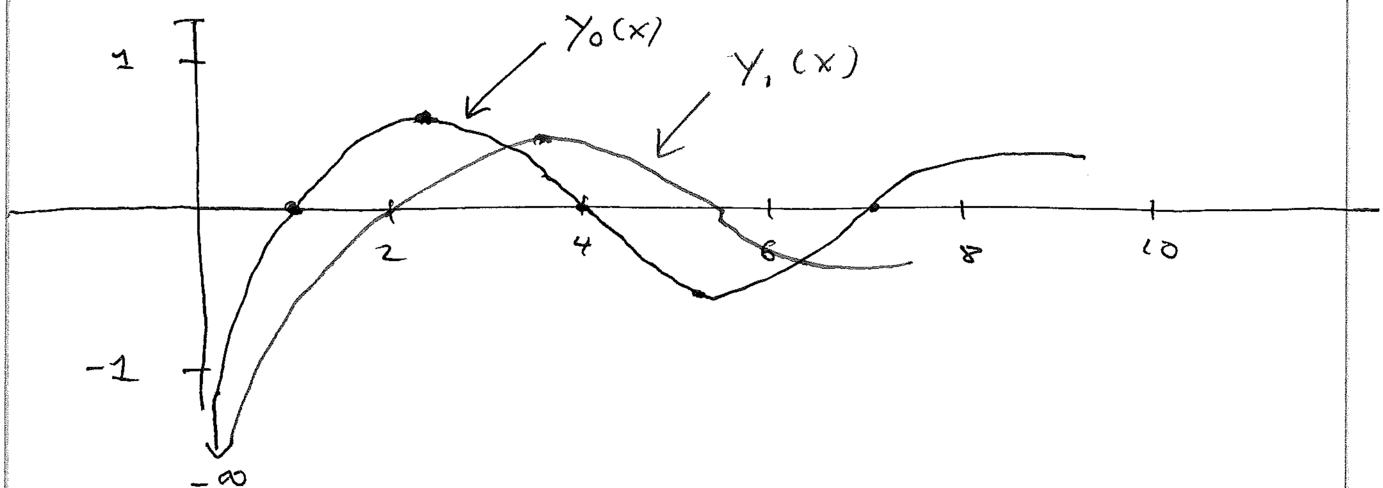

$m = 0, 1, 2, \dots$

$$\boxed{\begin{aligned} J_m(x) &\underset{x \gg 1}{\cong} \sqrt{\frac{2}{\pi x}} \cos\left[x - \frac{m\pi}{2} - \frac{\pi}{4}\right] \\ Y_m(x) &\underset{x \gg 1}{\cong} \sqrt{\frac{2}{\pi x}} \sin\left[x - \frac{m\pi}{2} - \pi/4\right] \end{aligned}}$$

So J and Y are linearly independent like cos and sin.

Numerical Root Finders ca be used to find $J_m(x_i^{(m)}) = 0$

$$Y_m(x_j^{(m)}) = 0$$

The roots which correspond to quantum numbers in cylindrical cavities.

$x << 1$ $\quad J_0$ has no root at $x=0$

$$J_m(0) = 0 \quad \forall\, m = 1, 2, 3, \ldots$$

$$Y_m(0) = -\infty \quad \forall\, n = 0, 1, 2, \ldots$$

$x >> 1$

$$J_m(x_i^{(m)}) \cong \cos\left[x - \frac{m\pi}{2} - \frac{\pi}{4}\right] = 0$$

$$\Rightarrow \quad x - \frac{m\pi}{2} - \frac{\pi}{4} = \frac{\pi}{2};\ \frac{3\pi}{2};\ \left[\frac{2i+1}{2}\right]\pi$$

$$\Rightarrow \quad x_i^{(m)} - \frac{m\pi}{2} - \frac{\pi}{4} \cong \left[\frac{2i+1}{2}\right]\pi$$

$$\Rightarrow \quad x_i^{(m)} \cong \left(\frac{2i+1}{2}\right)\pi + \frac{\pi}{2} + \frac{m\pi}{2} \qquad x_i^{(m)} >> 1$$

$$\cong \boxed{\left(\frac{2i+2+m}{2}\right)\pi}$$

Similarly roots of $Y_m(x_j^{(m)})$

$$\boxed{x_j^{(m)} \cong \left(\frac{2j+1+m}{2}\right)\pi}$$

alternate.