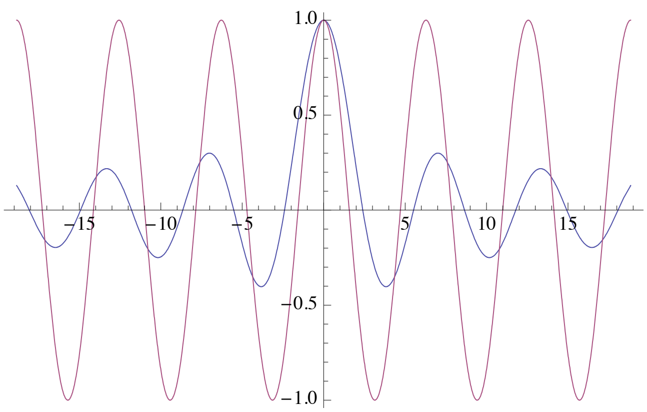
```
(*We expand J₀[x] and J₁[x] in a power series about x=
 0. It is easy to see that J₀[x] is an even function of x --
  like Cos[x] -- and J₁[x] is an odd function of x -- like Sin[x].*)

Series[BesselJ[0, x], {x, 0, 5}]
```

$$1 - \frac{x^2}{4} + \frac{x^4}{64} + O[x]^6$$

```
Series[BesselJ[1, x], {x, 0, 5}]
```

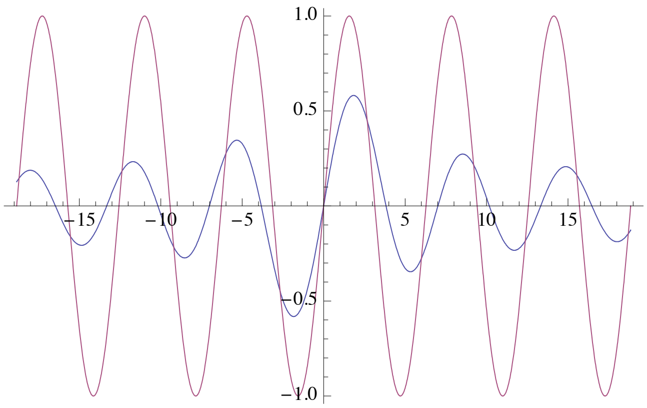
$$\frac{x}{2} - \frac{x^3}{16} + \frac{x^5}{384} + O[x]^6$$

```
(*Also like Cos[x] and Sin[x] we have that
 one is the negative of the derivative of the other.*)

D[Cos[x], x]

-Sin[x]

D[BesselJ[0, x], x]

-BesselJ[1, x]

(*We plot J₀[x] (blue) and Cos[x] (red) near x=0 and see J₀[x] is like a damped Cos[x]. *)

Plot[{BesselJ[0, x], Cos[x]}, {x, -6 Pi, 6 Pi}]
```




```
(*We plot J₁[x] (blue) and Sin[x] (red)  near x=0 and see J₁[x] is like a damped Sin[x]. *)
```

```
Plot[{BesselJ[1, x], Sin[x]}, {x, -6 Pi, 6 Pi}]
```



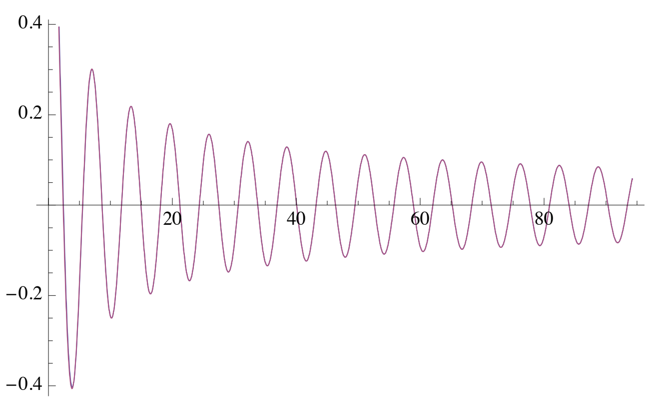

```
(*As proved in class, for |x|>>1 J0 is asymptotic to Sine and Cosine.*)
```

```
Series[BesselJ[0, x], {x, Infinity, 1}]
```

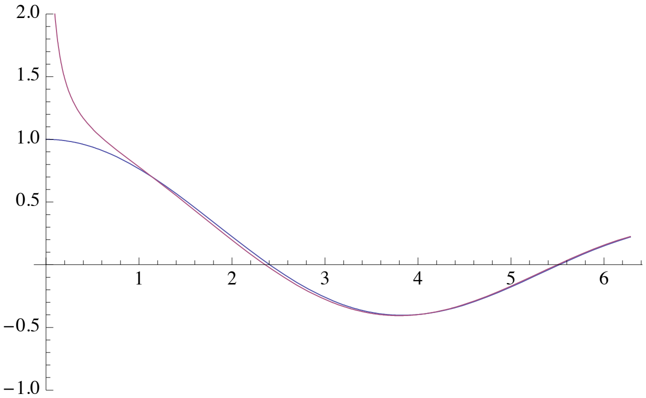
$$\text{Cos}\left[\frac{\pi}{4} - x\right] \left(\frac{\sqrt{\frac{2}{\pi}}}{\sqrt{x}} + O\left[\frac{1}{x}\right]^{3/2}\right) + \left(-\frac{1}{4\sqrt{2\pi}\, x^{3/2}} + O\left[\frac{1}{x}\right]^{2}\right) \text{Sin}\left[\frac{\pi}{4} - x\right]$$

```
(*We plot J0 for |x|>>
 1 and the leading term in the asymptotic expansion and see agreement is good.*)
```

$$\text{Plot}\left[\left\{\text{BesselJ[0, x], Cos}\left[\frac{\pi}{4} - x\right] * \text{Sqrt[2 / (Pi * x)]}\right\}, \{x, 0, 30\,\text{Pi}\}\right]$$



```
Plot[{BesselJ[0, x], Cos[π/4 - x] * Sqrt[2 / (Pi * x)]}, {x, 0, 2 Pi}, PlotRange -> {-1, 2}]
```

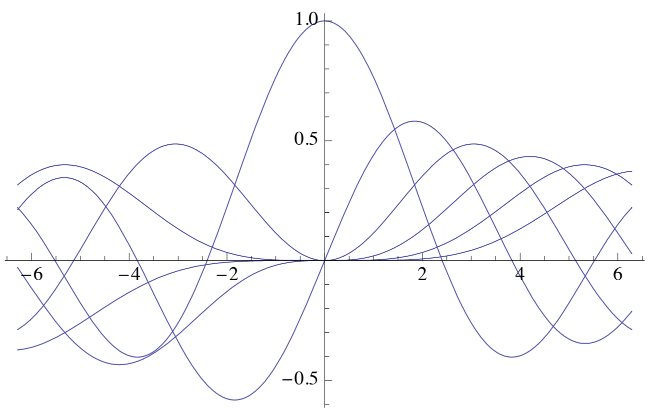


```
(*Hence we can use the asymptotic expression when |x|≥1 with good results.*)
```

In[1]:=
```
(*Here are plots of the first five Bessel functions of non-negative integer order.*)
```

In[10]:=
```
Plot[{Table[BesselJ[n, x], {n, 0, 5}]}, {x, -2 Pi, 2 Pi}]
```

Out[10]=

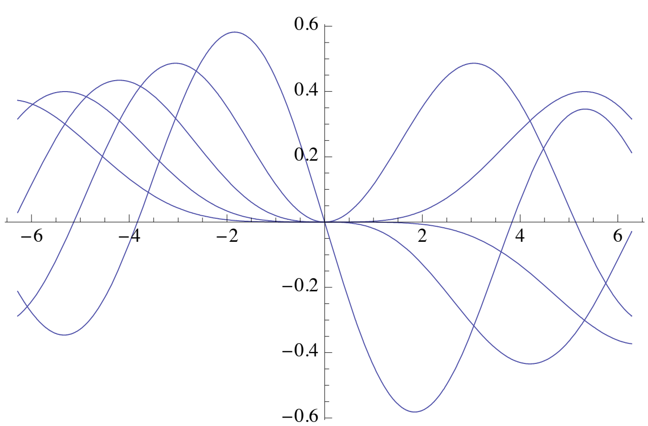


(*Note that all of them except J_0 vanish at the origin and alternate even and odd.*)

In[7]:=
```
(*Here we plot the first five Bessel functions of negative integer order.*)
```

In[11]:=
```
Plot[{Table[BesselJ[-n, x], {n, 1, 5}]}, {x, -2 Pi, 2 Pi}]
```

Out[11]=



```
(*Again all vanish at the origin and alternate even and odd.*)
```