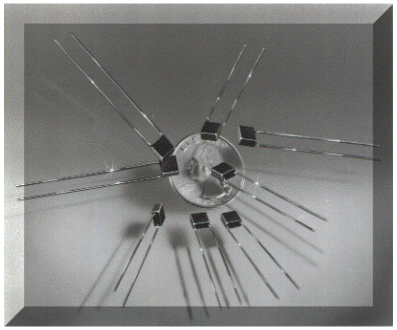
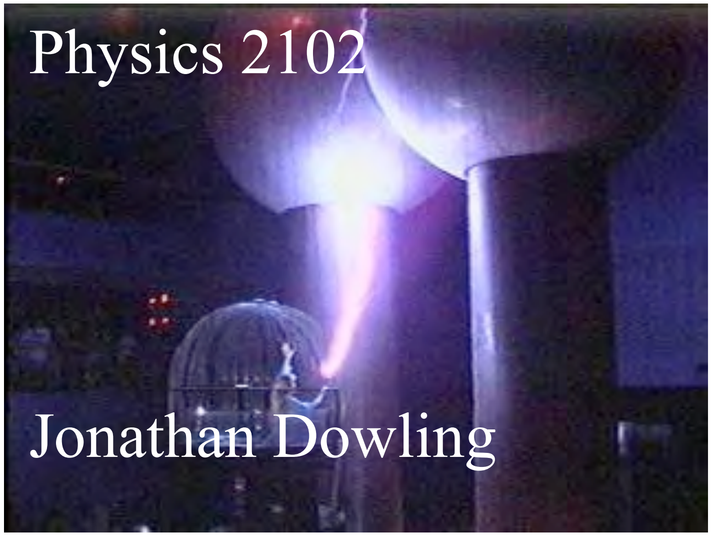
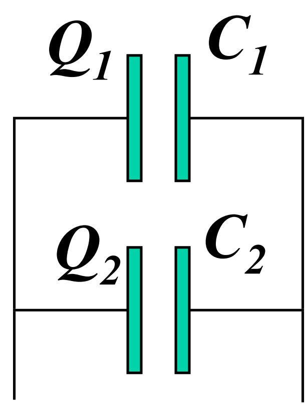


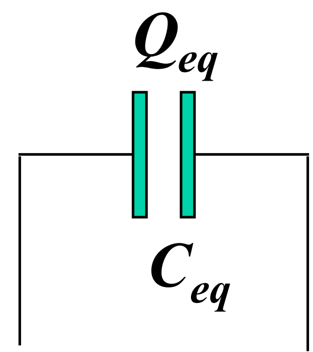


Physics 2102
Lecture 8
Capacitors II

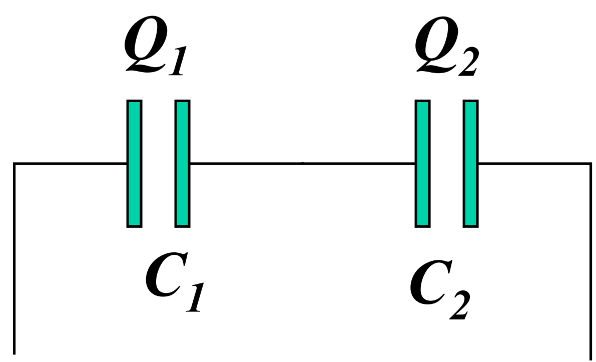
Capacitors in parallel and in series

- **In parallel :**
  - $C_{eq} = C_1 + C_2$
  - $V_{eq}=V_1=V_2$
  - $Q_{eq}=Q_1+Q_2$





- **In series :**
  - $1/C_{eq} = 1/C_1 + 1/C_2$
  - $V_{eq}=V_1+V_2$
  - $Q_{eq}=Q_1=Q_2$



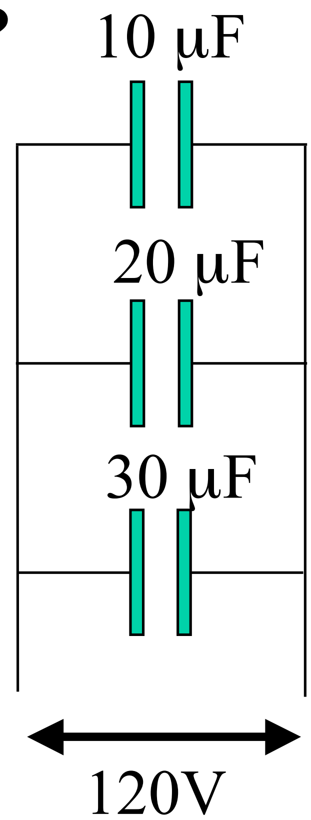
Example 1

What is the charge on each capacitor?

- $Q = CV$; $V = 120$ V
- $Q_1 = (10\ \mu F)(120V) = 1200\ \mu C$
- $Q_2 = (20\ \mu F)(120V) = 2400\ \mu C$
- $Q_3 = (30\ \mu F)(120V) = 3600\ \mu C$

Note that:

- Total charge (7200 μC) is shared between the 3 capacitors in the ratio $C_1{:}C_2{:}C_3$ — i.e. 1:2:3

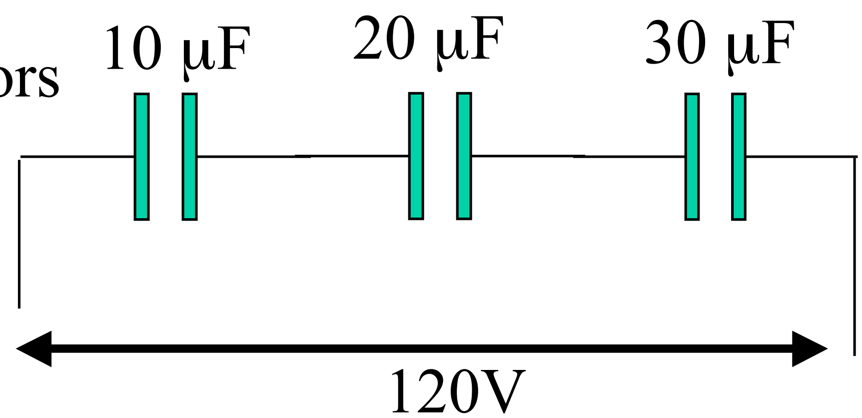


Example 2

What is the potential difference across each capacitor?

- $Q = CV$; Q is same for all capacitors
- Combined C is given by:

$$\frac{1}{C_{eq}} = \frac{1}{(10\mu F)} + \frac{1}{(20\mu F)} + \frac{1}{(30\mu F)}$$

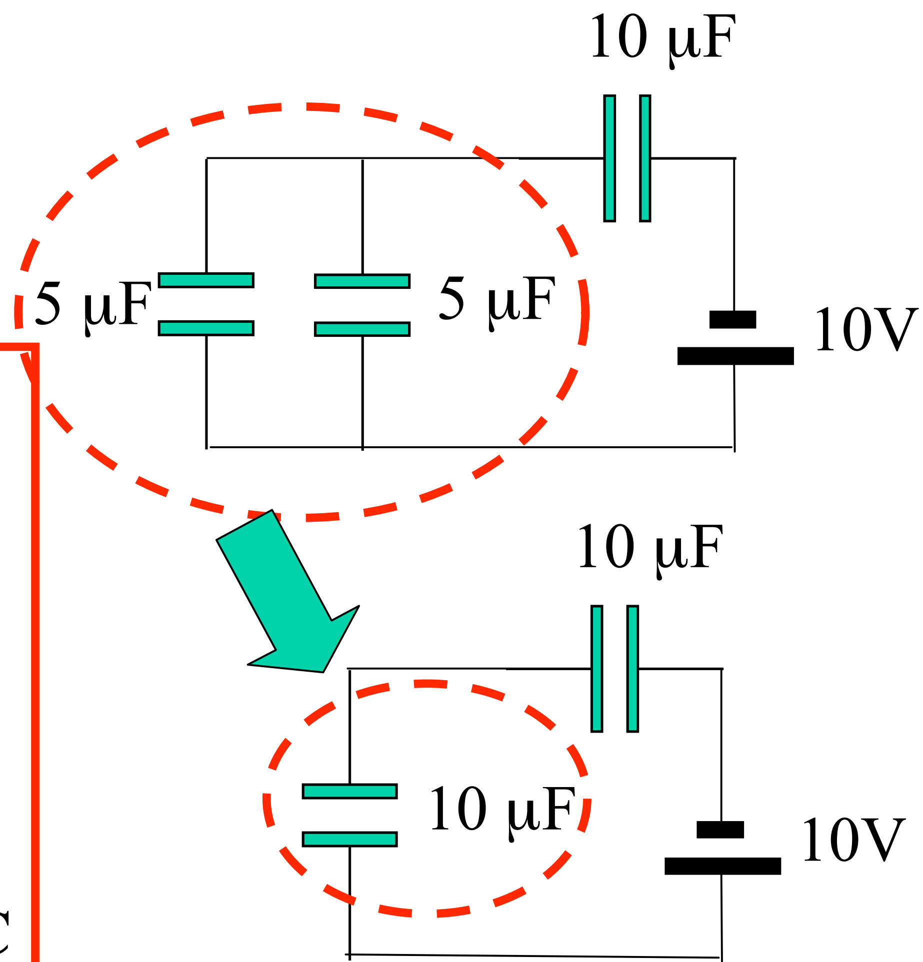


- $C_{eq} = 5.46\ \mu F$
- $Q = CV = (5.46\ \mu F)(120V) = 655\ \mu C$
- $V_1 = Q/C_1 = (655\ \mu C)/(10\ \mu F) = 65.5\ V$
- $V_2 = Q/C_2 = (655\ \mu C)/(20\ \mu F) = 32.75\ V$
- $V_3 = Q/C_3 = (655\ \mu C)/(30\ \mu F) = 21.8\ V$

Note: 120V is shared in the ratio of INVERSE capacitances i.e.1:(1/2):(1/3) (largest C gets smallest V)

Example 3

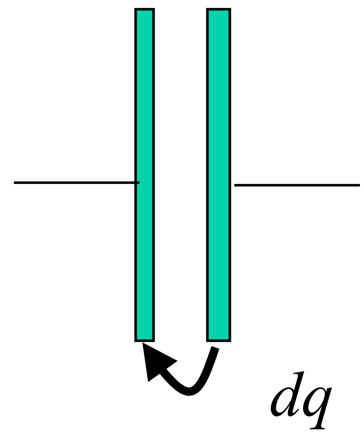
In the circuit shown, what is the charge on the 10μF capacitor?



- The two **5μF** capacitors are in parallel
- Replace by **10μF**
- Then, we have two **10μF** capacitors in series
- So, there is 5V across the **10μF** capacitor of interest
- Hence, Q = (**10μF**)(5V) = **50μC**

Energy Stored in a Capacitor

- Start out with uncharged capacitor
- Transfer small amount of charge dq from one plate to the other until charge on each plate has magnitude *Q*
- How much work was needed?



$$U = \int_0^Q V\,dq = \int_0^Q \frac{q}{C}\,dq = \frac{Q^2}{2C} = \frac{CV^2}{2}$$

Energy Stored in Electric Field

- **Energy stored in capacitor:** $U = Q^2/(2C) = CV^2/2$
- **View the energy as stored in ELECTRIC FIELD**
- **For example, parallel plate capacitor:**
  Energy DENSITY = energy/volume = u **=**

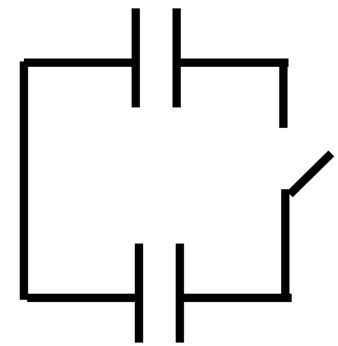
$$U = \frac{Q^2}{2C\,Ad} = \frac{Q^2}{2\left(\frac{\varepsilon_0 A}{d}\right)Ad} = \frac{Q^2}{2\varepsilon_0 A^2} = \frac{\varepsilon_0}{2}\left(\frac{Q}{\varepsilon_0 A}\right)^2 = \frac{\varepsilon_0 E^2}{2}$$

volume = Ad

General expression for any region with vacuum (or air)

Example

- **10μF capacitor is initially charged to 120V. 20μF capacitor is initially uncharged.**
- **Switch is closed, equilibrium is reached.**
- **How much energy is dissipated in the process?**



Initial charge on 10μF = (10μF)(120V)= 1200μC

After switch is closed, let charges = Q_1 and Q_2.

Charge is conserved: $Q_1 + Q_2 = 1200\mu C$

Also, V_{final} is same: $\frac{Q_1}{C_1} = \frac{Q_2}{C_2} \Rightarrow Q_1 = \frac{Q_2}{2} \Rightarrow$

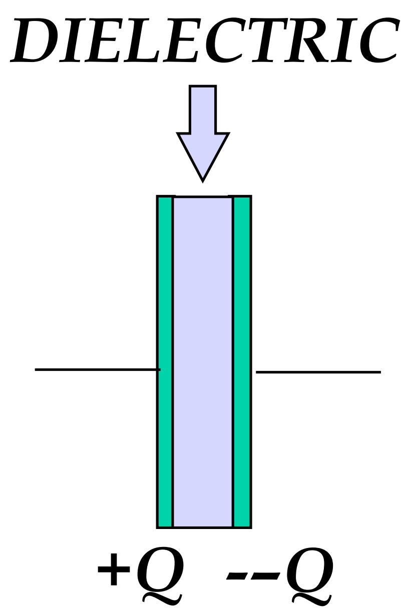
- $Q_1 = 400\mu C$
- $Q_2 = 800\mu C$
- $V_{final} = Q_1/C_1 = 40$ V

Initial energy stored = $(1/2)C_1V_{initial}^2 = (0.5)(10\mu F)(120)^2 = 72mJ$

Final energy stored = $(1/2)(C_1 + C_2)V_{final}^2 = (0.5)(30\mu F)(40)^2 = 24mJ$

Energy lost (dissipated) = 48mJ

Dielectric Constant

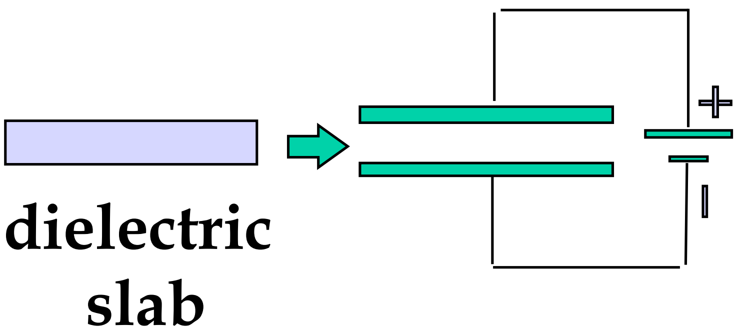


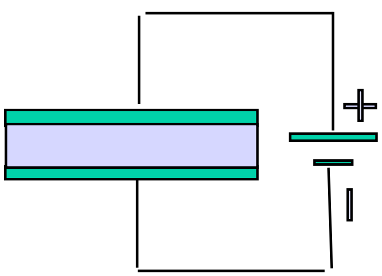
$$C = \kappa \varepsilon_0 A/d$$

- If the space between capacitor plates is filled by a dielectric, the capacitance **INCREASES** by a factor κ
- This is a useful, working definition for dielectric constant.
- Typical values of κ are 10–200

Example

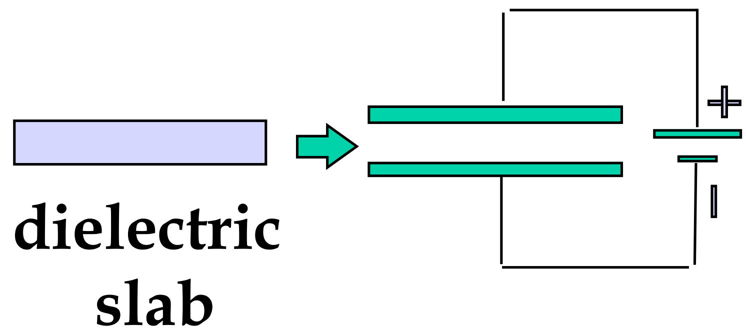
- Capacitor has charge *Q*, voltage *V*
- Battery remains connected while dielectric slab is inserted.
- Do the following increase, decrease or stay the same:
  - Potential difference?
  - Capacitance?
  - Charge?
  - Electric field?

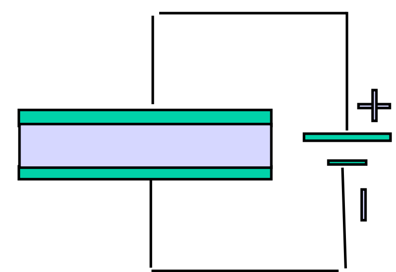




Example

- Initial values:
  capacitance = C; charge = Q;
  potential difference = V;
  electric field = E;
- Battery remains connected
- V is FIXED; $V_{new} = V$ (same)
- $C_{new} = \kappa C$ (increases)
- $Q_{new} = (\kappa C)V = \kappa Q$ (increases).
- Since $V_{new} = V$, $E_{new} = E$ (same)





Energy stored? $u=\varepsilon_0 E^2/2$ => $u=\kappa\varepsilon_0 E^2/2 = \varepsilon E^2/2$

Summary

- Any two charged conductors form a capacitor.
- Capacitance : $C = Q/V$

- Simple Capacitors:

    Parallel plates: $C = \varepsilon_0 A/d$
    Spherical : $C = 4\pi \varepsilon_0 ab/(b-a)$
    Cylindrical: $C = 2\pi \varepsilon_0 L/\ln(b/a)$

- Capacitors in series: same charge, not necessarily equal potential; equivalent capacitance $1/C_{eq}=1/C_1+1/C_2+\ldots$
- Capacitors in parallel: same potential; not necessarily same charge; equivalent capacitance $C_{eq}=C_1+C_2+\ldots$

- Energy in a capacitor: $U=Q^2/2C=CV^2/2$; energy density $u=\varepsilon_0 E^2/2$

- Capacitor with a dielectric: capacitance increases $C'=\kappa C$