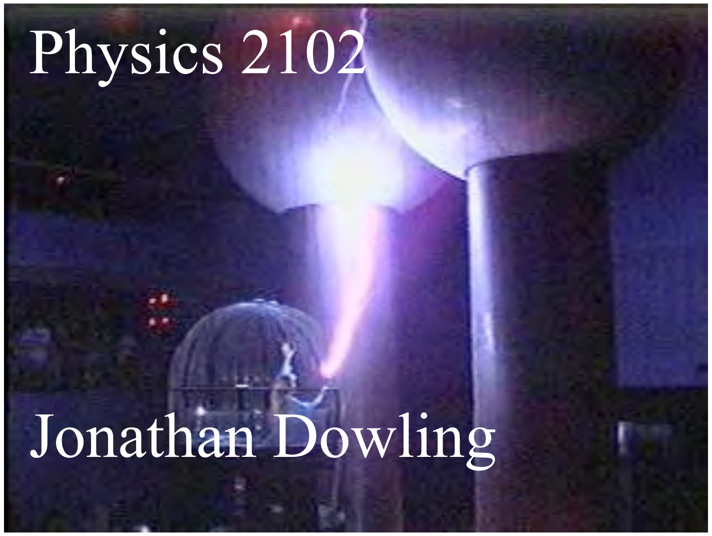
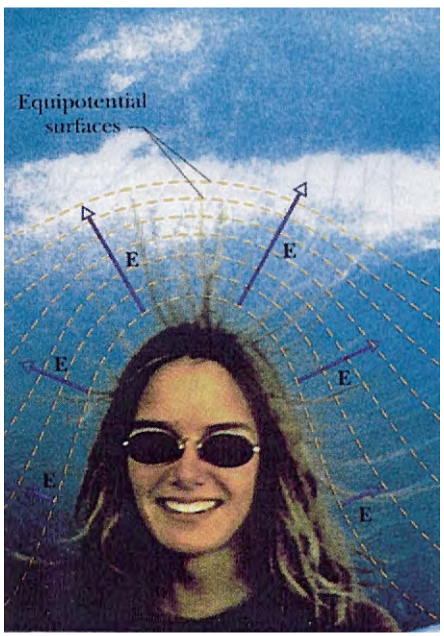
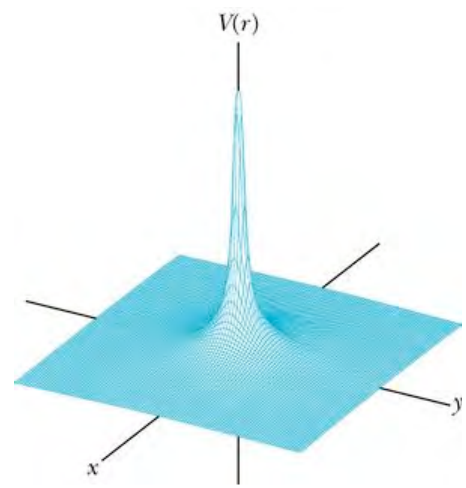


Physics 2102
Lecture 5

Electric Potential I





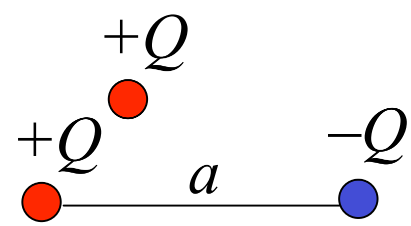
Electric potential energy

Electric potential energy of a system is equal to **minus** the work done by electrostatic forces when building the system (assuming charges were initially infinitely separated)

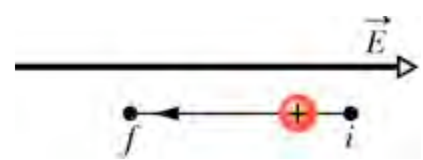
$$U = -W_\infty$$

The change in potential energy between an initial and final configuration is equal to **minus** the work done by the electrostatic forces:

$$\Delta U = U_f - U_i = -W$$



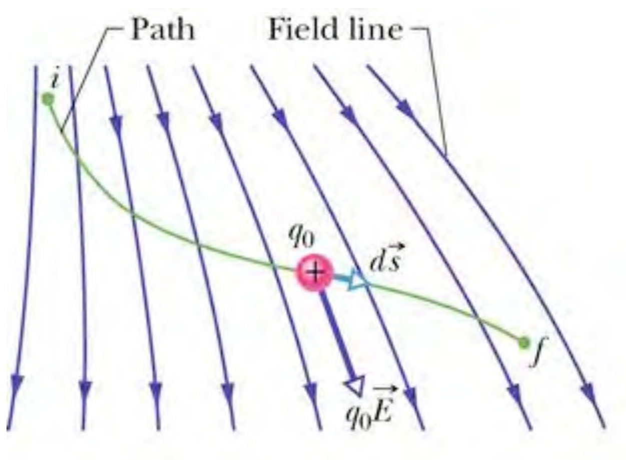
- What is the potential energy of a single charge?
- What is the potential energy of a dipole?
- A proton moves from point *i* to point *f* in a uniform electric field, as shown.
    - Does the electric field do positive or negative work on the proton?
    - Does the electric potential energy of the proton increase or decrease?



Electric potential

Electric potential difference between two points = work per unit charge needed to move a charge between the two points:

$$\Delta V = V_f - V_i = -W/q = \Delta U/q$$



$$dW = \vec{F} \bullet d\vec{s}$$

$$dW = q_0 \vec{E} \bullet d\vec{s}$$

$$W = \int_i^f dW = \int_i^f q_0 \vec{E} \bullet d\vec{s}$$

$$\Delta V = V_f - V_i = -\frac{W}{q_0} = -\int_i^f \vec{E} \bullet d\vec{s}$$

Electric potential energy, electric potential

Units : [U] = [W]=Joules;

[V]=[W/q] = Joules/C= Nm/C= Volts

[E]= N/C = Vm

1eV = work needed to move an electron through a potential difference of 1V:

$W=q\Delta V = e$ x 1V

= 1.60 10^{-19} C x 1J/C = 1.60 10^{-19} J

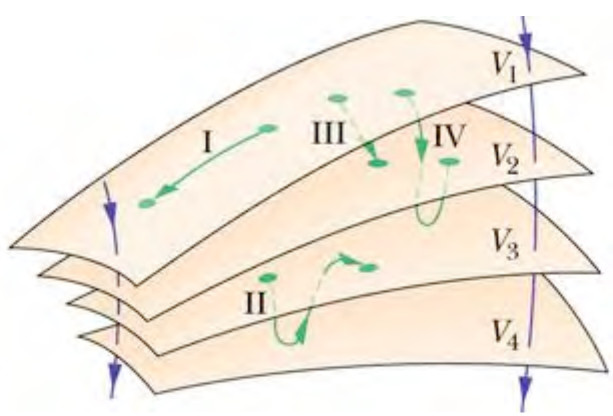
Equipotential surfaces

$$\Delta V = V_f - V_i = -\frac{W}{q_0} = -\int_i^f \vec{E} \bullet d\vec{s}$$

Given a charged system, we can:

• draw electric field lines: the electric field is tangent to the field lines

• draw equipotential surfaces: the electric potential is constant on the surface

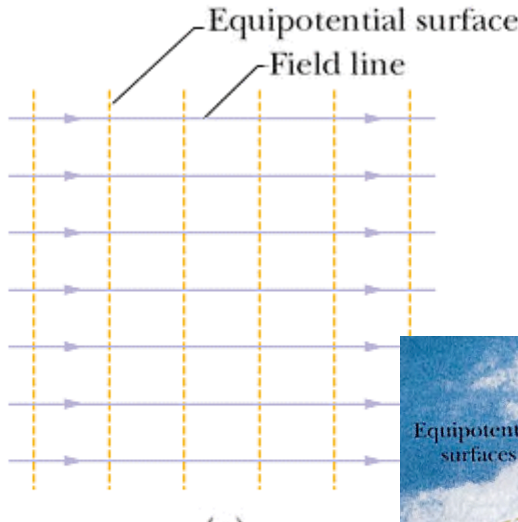


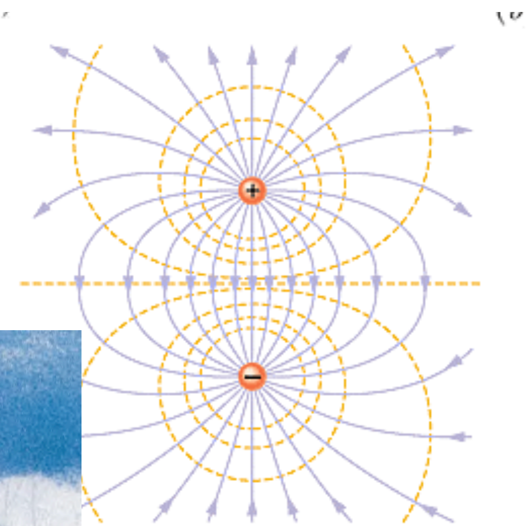
• Equipotential surfaces are perpendicular to electric field lines. Why??

• No work is needed to move a charge along an equipotential surface. Why??

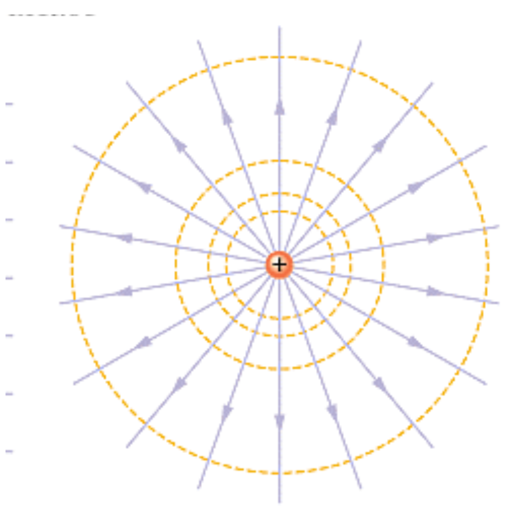
• Electric field lines always point towards equipotential surfaces with lower potential. Why??

Electric field lines and equipotential surfaces





(c)



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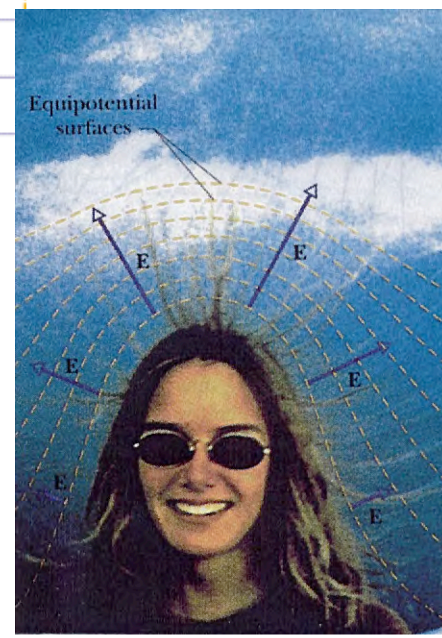


http://www.cco.caltech.edu/~phys1/java/phys1/EField/EField.html

Electric potential and electric potential energy

The change in potential energy of a charge q moving from point *i* to point *f* is equal to the work done by the applied force, which is equal to minus the work done by the electric field, which is related to the difference in electric potential:

$$\Delta U = U_f - U_i = W_{app} = -W = q\Delta V$$

We move a proton from point *i* to point *f* in a uniform electric field, as shown.

- Does the electric field do positive or negative work on the proton?
- Does the electric potential energy of the proton increase or decrease?
- Does our force do positive or negative work ?
- Does the proton move to a higher or lower potential?

Example

Consider a positive and a negative charge, freely moving in a uniform electric field. True or false?

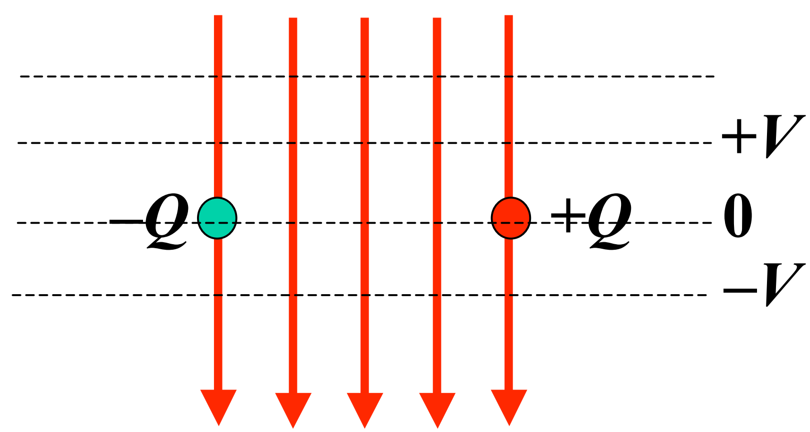
(a) Positive charge moves to points with lower potential.

(b) Negative charge moves to points with lower potential.

(c) Positive charge moves to a lower potential energy position.

(d) Negative charge moves to a lower potential energy position

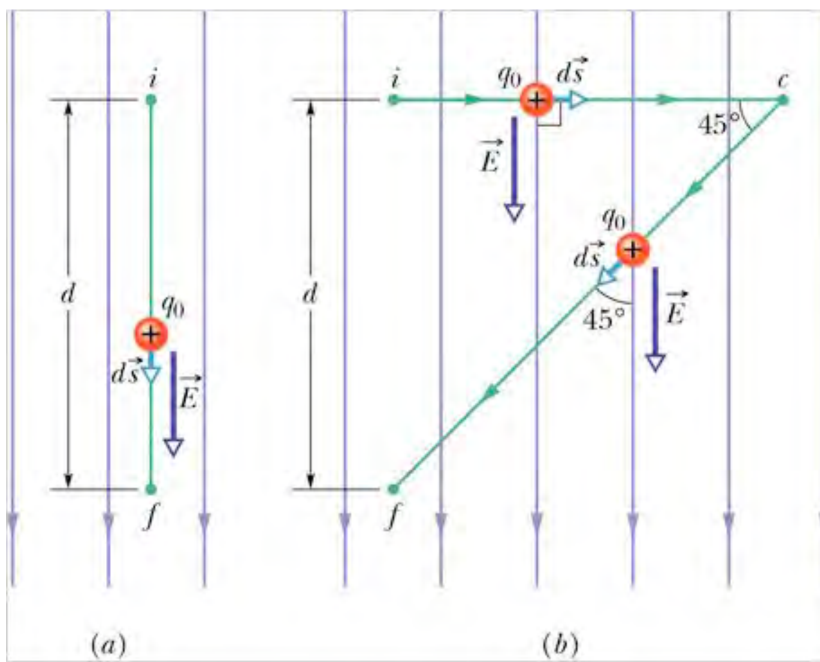
(a) True
(b) False
(c) True
(d) True



Conservative forces

The potential difference between two points is independent of the path taken to calculate it: electric forces are "conservative".

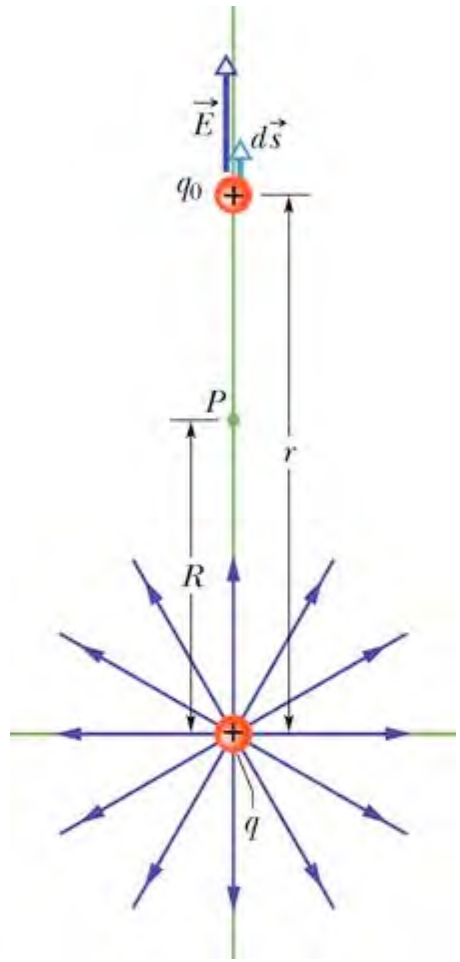
$$\Delta V = V_f - V_i = -\frac{W}{q_0} = \frac{\Delta U}{q_0} = -\int_i^f \vec{E} \bullet d\vec{s}$$



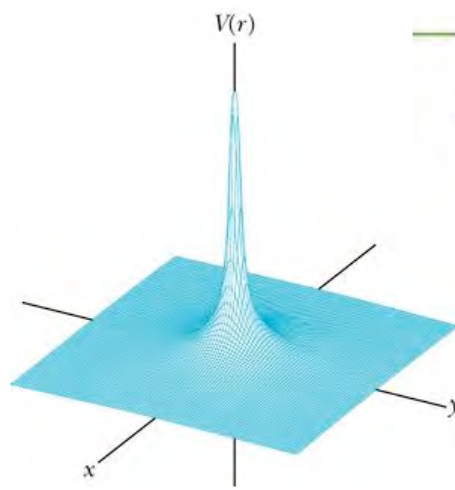
Electric Potential of a Point Charge

$$V = -\int_i^f \vec{E} \cdot d\vec{s} = -\int_\infty^P E\, ds =$$

$$= -\int_\infty^R \frac{kQ}{r^2}\, dr = +\frac{kQ}{r}\bigg|_\infty^R = +\frac{kQ}{R}$$

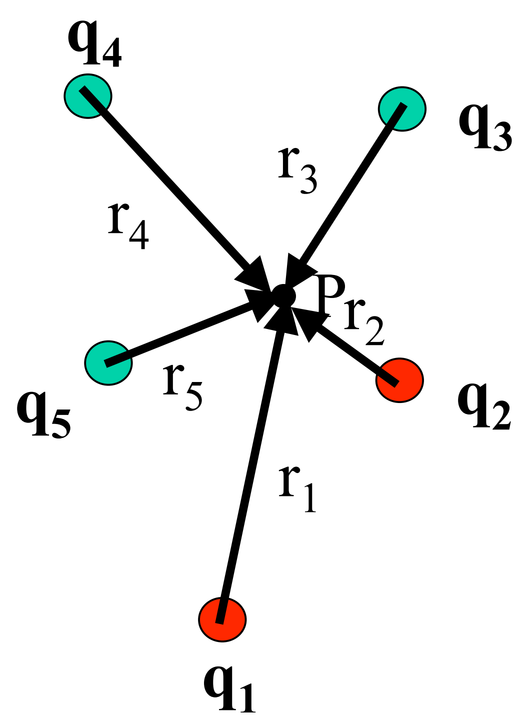


Note: if Q were a negative charge, V would be negative



Electric Potential of Many Point Charges

- Electric potential is a SCALAR not a vector.
- Just calculate the potential due to each individual point charge, and add together! (Make sure you get the SIGNS correct!)

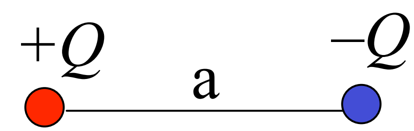


$$V = \sum_i k \frac{q_i}{r_i}$$

Electric potential and electric potential energy

$$\Delta U = W_{app} = q\Delta V$$

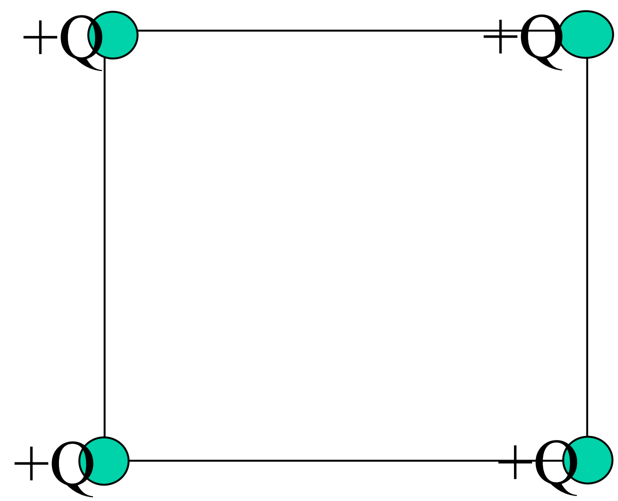
What is the potential energy of a dipole?

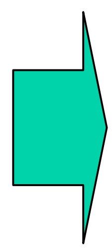


• First bring charge $+Q$: no work involved, no potential energy.
• The charge $+Q$ has created an electric potential everywhere, $V(r) = kQ/r$
• The work needed to bring the charge $-Q$ to a distance a from the charge $+Q$ is
$W_{app} = U = (-Q)V = (-Q)(+kQ/a) = -kQ^2/a$
• The dipole has a negative potential energy equal to $-kQ^2/a$: we had to do negative work to build the dipole (and the electric field did positive work).

Potential Energy of A System of Charges

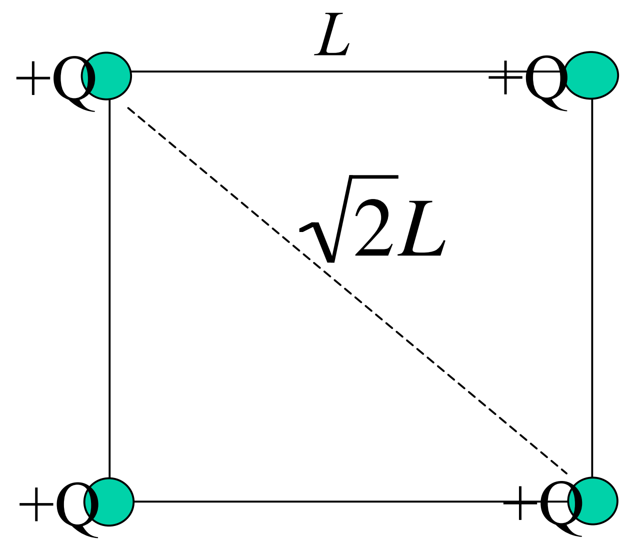
- 4 point charges (each +Q and equal mass) are connected by strings, forming a square of side L
- If all four strings suddenly snap, what is the kinetic energy of each charge when they are very far apart?
- Use conservation of energy:
  - Final kinetic energy of all four charges = initial potential energy stored = energy required to assemble the system of charges





Do this from scratch!

Potential Energy of A System of Charges: Solution



- No energy needed to bring in first charge: $U_1=0$
- Energy needed to bring in 2nd charge: $$U_2 = QV_1 = \frac{kQ^2}{L}$$
- Energy needed to bring in 3rd charge =

$$U_3 = QV = Q(V_1 + V_2) = \frac{kQ^2}{L} + \frac{kQ^2}{\sqrt{2}L}$$

- Energy needed to bring in 4th charge =

$$U_4 = QV = Q(V_1 + V_2 + V_3) = \frac{2kQ^2}{L} + \frac{kQ^2}{\sqrt{2}L}$$

Total potential energy is sum of all the individual terms shown on left hand side = $\frac{kQ^2}{L}\left(4+\sqrt{2}\right)$

So, final kinetic energy of each charge = $\frac{kQ^2}{4L}\left(4+\sqrt{2}\right)$

Summary:

- **Electric potential**: work needed to bring +1C from infinity; units V = Volt
- Electric potential uniquely defined for every point in space -- independent of path!
- Electric potential is a **scalar** — add contributions from individual point charges
- We calculated the electric potential produced by a single charge: $V=kq/r$, and by continuous charge distributions : $V=\int kdq/r$
- **Electric potential energy**: work used to build the system, charge by charge. Use $W=qV$ for each charge.