Physics 2102

Exam 2: Review Session
CH 24–28

Some links on exam stress:
http://appl003.lsu.edu/slas/cas.nsf/$Content/Stress+Management+Tip+1
http://wso.williams.edu/orgs/peerh/stress/exams.html
http://www.thecalmzone.net/Home/ExamStress.php
http://www.staithes.demon.co.uk/exams.html
Exam 2

- (Ch24) Sec.11 (Electric Potential Energy of a System of Point Charges); Sec.12 (Potential of Charged Isolated Conductor)
- (Ch 25) Capacitors: capacitance and capacitors; caps in parallel and in series, dielectrics; energy, field and potential in capacitors.
- (Ch 26) Current and Resistance: current, current density and drift velocity; resistance and resistivity; Ohm’s law.
- (Ch 27) Circuits: emf devices, loop and junction rules; resistances in series and parallel; DC single and multiloop circuits, power; RC circuits.
Potential Energy of a System of Charges
Potential Energy of A System of Charges

- 4 point charges (each $+Q$) are connected by strings, forming a square of side $L$
- If all four strings suddenly snap, what is the kinetic energy of each charge when they are very far apart?
- Use conservation of energy:
  - Final kinetic energy of all four charges
  - $= \text{initial potential energy stored} = \text{energy required to assemble the system of charges}$

Do this from scratch! Don’t memorize the formula in the book! We will change the numbers!!!
Potential Energy of A System of Charges: Solution

• No energy needed to bring in first charge: \( U_1 = 0 \)

• Energy needed to bring in 2nd charge:
  \[
  U_2 = QV_1 = \frac{kQ^2}{L}
  \]

• Energy needed to bring in 3rd charge =
  \[
  U_3 = QV = Q(V_1 + V_2) = \frac{kQ^2}{L} + \frac{kQ^2}{\sqrt{2}L}
  \]

• Energy needed to bring in 4th charge =
  \[
  U_4 = QV = Q(V_1 + V_2 + V_3) = \frac{2kQ^2}{L} + \frac{kQ^2}{\sqrt{2}L}
  \]

Total potential energy is sum of all the individual terms shown on left hand side = \( \frac{kQ^2}{L} \left( 4 + \sqrt{2} \right) \)

So, final kinetic energy of each charge = \( \frac{kQ^2}{4L} \left( 4 + \sqrt{2} \right) \)
Electric Field and Potential in and around a Charged Conductor: A Summary

1. All the charges reside on the conductor surface.
2. The electric field inside the conductor is zero: \( E_{\text{in}} = 0 \).
3. The electric field just outside the conductor is: \( E_{\text{out}} = \frac{\sigma}{\varepsilon_0} \).
4. The electric field just outside the conductor is perpendicular to the conductor surface.
5. All the points on the surface and inside the conductor have the same potential. The conductor is an equipotential surface.
Capacitors

\[ E = \frac{\sigma}{\varepsilon_0} = \frac{q}{A\varepsilon_0} \]
\[ E = V \frac{d}{d} \]
\[ q = CV \]
\[ C = \varepsilon_0 \frac{A}{d} \]
\[ C = \kappa \varepsilon_0 \frac{A}{d} \]
\[ C = \varepsilon_0 \frac{ab}{(b-a)} \]
Current and resistance

\[ i = \frac{dq}{dt} \]

Junction rule

\[ V = iR \]

\[ E = J\rho \]

\[ R = \frac{\rho L}{A} \]

\[ \rho = \rho_0(1 + \alpha(T - T_0)) \]
DC Circuits

Single loop

\[ V = iR \]

\[ P = iV \]

Multiloop

Loop rule

Real battery

Potential (V)

Emf device

Resistor
Resistors

**Key formula:** \( V = iR \)

**In series:** same current

\[ R_{eq} = \sum R_j \]

**In parallel:** same voltage

\[ \frac{1}{R_{eq}} = \sum \frac{1}{R_j} \]

Capacitors

**Key formula:** \( Q = CV \)

**In series:** same charge

\[ \frac{1}{C_{eq}} = \sum \frac{1}{C_j} \]

**In parallel:** same voltage

\[ C_{eq} = \sum C_j \]
Capacitors and Resistors in Series and in Parallel

- What’s the equivalent resistance (capacitance)?
- What’s the current (charge) in each resistor (capacitor)?
- What’s the potential across each resistor (capacitor)?
- What’s the current (charge) delivered by the battery?
**RC Circuits**

Time constant: $RC$

Charging: $q(t) = CE \left(1 - e^{-t/RC} \right)$

Discharging: $q(t) = q_0 e^{-t/RC}$

$i(t) = dq/dt$
Capacitors: Checkpoints, Questions
Problem 25-21

When switch S is thrown to the left, the plates of capacitor 1 acquire a potential $V_0$. Capacitors 2 and 3 are initially uncharged. The switch is now thrown to the right. What are the final charges $q_1$, $q_2$, and $q_3$ on the capacitors?
21. The charges on capacitors 2 and 3 are the same, so these capacitors may be replaced by an equivalent capacitance determined from

\[
\frac{1}{C_{\text{eq}}} = \frac{1}{C_2} + \frac{1}{C_3} = \frac{C_2 + C_3}{C_2 C_3}.
\]

Thus, \( C_{\text{eq}} = \frac{C_2 C_3}{C_2 + C_3} \). The charge on the equivalent capacitor is the same as the charge on either of the two capacitors in the combination and the potential difference across the equivalent capacitor is given by \( q_2/C_{\text{eq}} \). The potential difference across capacitor 1 is \( q_1/C_1 \), where \( q_1 \) is the charge on this capacitor. The potential difference across the combination of capacitors 2 and 3 must be the same as the potential difference across capacitor 1, so \( q_1/C_1 = q_2/C_{\text{eq}} \). Now some of the charge originally on capacitor 1 flows to the combination of 2 and 3. If \( q_0 \) is the original charge, conservation of charge yields \( q_1 + q_2 = q_0 = C_1 V_0 \), where \( V_0 \) is the original potential difference across capacitor 1.
(a) Solving the two equations

\[
\frac{q_1}{C_1} = \frac{q_2}{C_{eq}} \quad \text{and} \quad q_1 + q_2 = C_1V_0
\]

for \( q_1 \) and \( q_2 \), we obtain

\[
q_1 = \frac{C_1^2V_0}{C_{eq} + C_1} = \frac{C_1^2V_0}{\frac{C_2C_3}{C_2 + C_3} + C_1} = \frac{C_1^2(C_2 + C_3)V_0}{C_1C_2 + C_1C_3 + C_2C_3}.
\]

With \( V_0 = 12.0 \text{ V} \), \( C_1 = 4.00 \text{ } \mu\text{F} \), \( C_2 = 6.00 \text{ } \mu\text{F} \) and \( C_3 = 3.00 \text{ } \mu\text{F} \), we find \( C_{eq} = 2.00 \text{ } \mu\text{F} \) and \( q_1 = 32.0 \text{ } \mu\text{C} \).

(b) The charge on capacitors 2 is

\[
q_2 = C_1V_0 - q_1 = (4.00 \mu\text{F})(12.0\text{V}) - 32.0 \mu\text{F} = 16.0 \mu\text{F}
\]

(c) The charge on capacitor 3 is the same as that on capacitor 2:

\[
q_3 = C_1V_0 - q_1 = (4.00 \mu\text{F})(12.0\text{V}) - 32.0 \mu\text{F} = 16.0 \mu\text{F}
\]
Current and Resistance: Checkpoints, Questions
Problem 26-56

A cylindrical resistor of radius 5.0mm and length 2.0 cm is made of a material that has a resistivity of $3.5 \times 10^{-5}$ $\Omega$m. What are the (a) current density and (b) the potential difference when the energy dissipation rate in the resistor is 1.0W?
56. (a) Since $P = i^2 R = J^2 A^2 R$, the current density is

$$J = \frac{1}{A} \sqrt{\frac{P}{R}} = \frac{1}{A} \sqrt{\frac{P}{\rho L / A}} = \sqrt{\frac{P}{\rho L A}}$$

$$= \sqrt{\frac{1.0 \text{ W}}{\pi (3.5 \times 10^{-5} \Omega \cdot \text{m}) (2.0 \times 10^{-2} \text{ m}) (5.0 \times 10^{-3} \text{ m})^2}} = 1.3 \times 10^5 \text{ A} / \text{m}^2.$$ 

(b) From $P = iV = JAV$ we get

$$V = \frac{P}{AJ} = \frac{P}{\pi r^2 J} = \frac{1.0 \text{ W}}{\pi (5.0 \times 10^{-3} \text{ m})^2 (1.3 \times 10^5 \text{ A} / \text{m}^2)} = 9.4 \times 10^{-2} \text{ V}.$$
Problem: 27.P.018. [406649]

Figure 27-33 shows five 5.00 resistors.
(Hint: For each pair of points, imagine that a battery is connected across the pair.)

Fig. 27-33
(a) Find the equivalent resistance between points F and H.

(b) Find the equivalent resistance between points F and G.
18. (a) \( R_{eq} (FH) = (10.0 \, \Omega)(10.0 \, \Omega)(5.00 \, \Omega)/[(10.0 \, \Omega)(10.0 \, \Omega) + 2(10.0 \, \Omega)(5.00 \, \Omega)] = 2.50 \, \Omega. \)

(b) \( R_{eq} (FG) = (5.00 \, \Omega) R/(R + 5.00 \, \Omega) \), where

\[
R = 5.00 \, \Omega + (5.00 \, \Omega)(10.0 \, \Omega)/(5.00 \, \Omega + 10.0 \, \Omega) = 8.33 \, \Omega.
\]

So \( R_{eq} (FG) = (5.00 \, \Omega)(8.33 \, \Omega)/(5.00 \, \Omega + 8.33 \, \Omega) = 3.13 \, \Omega. \)
Problem: 27.P.046. [406629]

In an RC series circuit, $\mathcal{E} = 17.0$ V, $R = 1.50$ M$\Omega$, and $C = 1.80$ $\mu$F.

(a) Calculate the time constant.

(b) Find the maximum charge that will appear on the capacitor during charging.

(c) How long does it take for the charge to build up to 10.0 $\mu$C?
46. (a) \( \tau = RC = (1.40 \times 10^6 \, \Omega)(1.80 \times 10^{-6} \, \text{F}) = 2.52 \, \text{s.} \)

(b) \( q_o = \varepsilon C = (12.0 \, \text{V})(1.80 \, \mu \text{F}) = 21.6 \, \mu \text{C}. \)

(c) The time \( t \) satisfies \( q = q_o(1 - e^{-t/RC}) \), or

\[
t = RC \ln \left( \frac{q_o}{q_o - q} \right) = (2.52 \, \text{s}) \ln \left( \frac{21.6 \, \mu \text{C}}{21.6 \, \mu \text{C} - 16.0 \, \mu \text{C}} \right) = 3.40 \, \text{s}.
\]
Magnetic Forces and Torques

\[ \vec{F} = q \vec{v} \times \vec{B} + q \vec{E} \]

\[ d\vec{F} = i \, d\vec{L} \times \vec{B} \]

\[ \vec{\tau} = \vec{\mu} \times \vec{B} \]
Magnetic Torque on a Current Loop

Consider the rectangular loop in fig. (a) with sides of lengths \(a\) and \(b\) and that carries a current \(i\). The loop is placed in a magnetic field so that the normal \(\hat{n}\) to the loop forms an angle \(\theta\) with \(\vec{B}\). The magnitude of the magnetic force on sides 1 and 3 is \(F_1 = F_3 = iab \sin 90^\circ = iab\). The magnetic force on sides 2 and 4 is \(F_2 = F_4 = ibB \sin(90 - \theta) = ibB \cos \theta\). These forces cancel in pairs and thus \(F_{\text{net}} = 0\).

The torque about the loop center \(C\) of \(F_2\) and \(F_4\) is zero because both forces pass through point \(C\). The moment arm for \(F_1\) and \(F_3\) is equal to \((b / 2) \sin \theta\). The two torques tend to rotate the loop in the same (clockwise) direction and thus add up. The net torque \(\tau = \tau_1 + \tau_3 = (iabB / 2) \sin \theta + (iabB / 2) \sin \theta = iabB \sin \theta = iAB \sin \theta\). 

\(\text{(28-13)}\)
Magnetic Dipole Moment

The torque of a coil that has \( N \) loops exerted by a uniform magnetic field \( B \) and carries a current \( i \) is given by the equation \( \tau = N i A B \).

We define a new vector \( \vec{\mu} \) associated with the coil, which is known as the magnetic dipole moment of the coil.

\[
\vec{\tau} = \vec{\mu} \times \vec{B} \quad \text{and} \quad U = -\vec{\mu} \cdot \vec{B}
\]

The magnitude of the magnetic dipole moment is \( \mu = N i A \).

Its direction is perpendicular to the plane of the coil.

The sense of \( \vec{\mu} \) is defined by the right-hand rule. We curl the fingers of the right hand in the direction of the current. The thumb gives us the sense. The torque can be expressed in the form \( \tau = \mu B \sin \theta \) where \( \theta \) is the angle between \( \vec{\mu} \) and \( \vec{B} \).

In vector form: \( \vec{\tau} = \vec{\mu} \times \vec{B} \).

The potential energy of the coil is: \( U = -\mu B \cos \theta = -\vec{\mu} \cdot \vec{B} \).

\( U \) has a minimum value of \( -\mu B \) for \( \theta = 0 \) (position of stable equilibrium).

\( U \) has a maximum value of \( \mu B \) for \( \theta = 180^\circ \) (position of unstable equilibrium).

Note: For both positions the net torque is \( \tau = 0 \).
Ch 28: Checkpoints and Questions
Problem: 28.P.024. [566302]
In the figure below, a charged particle moves into a region of uniform magnetic field, goes through half a circle, and then exits that region. The particle is either a proton or an electron (you must decide which). It spends 160 ns in the region.

(a) What is the magnitude of B?

(b) If the particle is sent back through the magnetic field (along the same initial path) but with 3.00 times its previous kinetic energy, how much time does it spend in the field during this trip?
24. We consider the point at which it enters the field-filled region, velocity vector pointing downward. The field points out of the page so that $\vec{v} \times \vec{B}$ points leftward, which indeed seems to be the direction it is "pushed"; therefore, $q > 0$ (it is a proton).

(a) Eq. 28-17 becomes $T = \frac{2\pi m_p}{e|\vec{B}|}$, or

$$2 \left(130 \times 10^{-9}\right) = \frac{2\pi \left(1.67 \times 10^{-27}\right)}{\left(1.60 \times 10^{-19}\right)|\vec{B}|}$$

which yields $|\vec{B}| = 0.252$ T.

(b) Doubling the kinetic energy implies multiplying the speed by $\sqrt{2}$. Since the period $T$ does not depend on speed, then it remains the same (even though the radius increases by a factor of $\sqrt{2}$). Thus, $t = T/2 = 130$ ns, again.