ATOMS IN STRONG MAGNETIC FIELDS —
A "NEW" AREA OF LABORATORY ATOMIC PHYSICS RESEARCH
WITH IMPLICATIONS FOR ASTROPHYSICS AND SOLID-STATE PHYSICS

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We speak of atoms in strong magnetic fields \( B \) when the magnetic forces are comparable to or greater than Coulomb forces. Roughly speaking, this occurs \(^1\) for an isolated hydrogen atom when \( B \) exceeds \( B_0 \) where \( B_0 = \left( \frac{\mu^2 c^3}{K} \right)^{1/2} = 2.35 \times 10^6 \text{G} \), and \( n \) is the principal quantum number. Thus for \( B = B_0 \approx 2.35 \times 10^6 \text{G} \) (which we will take as a typical laboratory field), we are dealing with a strong magnetic field when we consider values of \( n \approx 50 \). This is a "new" area for laboratory atomic physics experiments. Actually, pioneering work in this area, both experimental \(^2\) and theoretical, \(^3\) was carried out as far back as 1939 but, except for some investigations pertaining to solid-state physics, the subject lay dormant for a long time. The recent discovery \(^4\) of strong magnetic fields in pulsars (\( B = 10^{12} \text{G} \)) and some white dwarfs (\( B = 10^6 - 10^8 \text{G} \)) led to a resurgence of interest in strong \( B \) fields. This interest has also been kindled by the work of Garton and Tomkins \(^5\) on the Ba I absorption spectrum in a magnetic field of \( 2.4 \times 10^6 \text{G} \). Their measurements extended to as high as \( n = 75 \).

For a theoretical analysis of the Ba I spectrum, we will consider the motion of the valence electron in a magnetic field \( B = B_2 \) and a Coulomb potential \(-Z_{\text{eff}} e/r\). Here \( Z_{\text{eff}} \) is the effective charge seen by the valence electron and its magnitude is actually a function of \( B \) — since the motion of all the electrons is affected by the magnetic field, the outer electrons being affected the most. Now if \( E(Z, B) \) denotes the energy of a hydrogen-like atom of atomic number \( Z \) in a magnetic field \( B \), then it follows \(^6\) rigorously that \( E(Z, B) = Z^2 E_1(B/Z^2) \), where \( B/Z^2 \). This relation enables us to confine our attention to the hydrogen atom.
Since the variational techniques used to obtain the low-lying H-atom levels\(^7\) are not suitable for a consideration of highly excited states, we resort to a modification of the adiabatic approximation.\(^3\) This approach is based on the fact that, when magnetic forces dominate over Coulomb forces, the motion in the x-y plane is very rapid and very weakly dependent on the presence of the Coulomb field while the relatively slow z-motion is determined by a one-dimensional potential, \(V(z)\), which is the average of the Coulomb potential over the very rapid motion in the x-y plane. Now in the absence of a Coulomb field, the energy of a nonrelativistic electron, \(E_m\) say, is given by\(^8\)

\[
E_m = n\hbar \omega ,
\]

where \(\omega\) is the cyclotron frequency, the magnetic quantum number \(n=0,1,2,\ldots\), and the Pauli spin contribution has been included. These levels are actually infinitely degenerate in the sense that \(n=\{n_\rho + (|m|-m+1)/2 + m_s\}\), where \(n_\rho=0,1,2,3,\ldots\), \(m=0,1,\pm2,\ldots\) is the z-component of the angular momentum, and \(m_s=\pm1/2\) denotes the spin quantum number. The corresponding classical radius\(^8\) is \(r_m = (2n B_o / B)^{1/2} a_o\), where \(a_o\) is the Bohr radius. The corresponding wave-function we denote by

\[
\psi_m = (2\pi)^{-1/2} e^{ikz} f(\rho,\phi) .
\]

In the presence of both magnetic and Coulomb fields, we write the energy \(E\) as

\[
E = E_m + E_C ,
\]

where \(E_C\) is the Coulomb field contribution, and the wave-function as

\[
\psi = f(z)F(\rho,\phi) .
\]

Thus

\[
V(z) = -e^2 <F|(\rho^2 + z^2)^{-1/2}|F> ,
\]

and the solution of the one-dimensional Schrödinger equation for a particle in this potential gives us both \(f(z)\) and \(E_C\). In general, this equation must be solved numerically\(^9\) but in certain limits analytic treatments are possible. Since the charge
probability density is sharply peaked at $\rho_m$ we may substitute $\rho_m$ for $\rho$ in Eq. (4). In addition, we note that the confinement of the atom in the z-direction is $-a_o$. Now, for low-lying states in a strong B field ($B \gg B_o$), $\rho_m \ll a_o$ and thus $(\rho_m^2 + z^2)^{-1/2}$ can be approximated by $(\rho_m + z)^{-1}$, which permits an analytic evaluation of $E_C$, giving the characteristic $\ln^2(\rho_m/a_o)$ behavior of the energy.

However, for highly-excited states in laboratory fields we have the reverse situation i.e. $\rho_m \gg a_o$. For example, for $B = B_L$, we have $\rho_m = 458 n_{1/2} a_o$. Hence we may take $E_C = -(e^2/\rho_m)$. Substituting in Eq. (2), this leads to the result

$$E = \hbar \omega [n - (2n B/B_o)^{-1/2}]$$

(5)

It follows that $\delta E/\delta n = 1.5\hbar \omega$ at $E = 0$, in agreement with the results of Ref. 5 for the $\sigma$ lines. We hope to present soon a comparison of the analytic result for $E_C$ with numerically calculated values of $E_C$.

Turning to implications for astrophysics, the effect of strong B fields on the Balmer spectra has been investigated in detail, particularly with a view to deducing the values of B found in magnetic white dwarfs. The energy levels of the two-electron systems, He I and H$^-$, has also been investigated. For small B fields, the singlet even-parity state is the lowest state of He I and the binding decreases with increasing B until we reach $B = 1.7 \times 10^9 G$. Beyond this cross-over point, the lowest energy state is a triplet odd-parity state and the binding energy increases with increasing B. For H$^-$ the behavior is similar except that the cross-over point is at $B = 1.2 \times 10^8 G$ and the system is actually unbound in the intermediate range $1.2 \times 10^8 G < B < 3.3 \times 10^8 G$.

In semiconductors, the Coulomb field is effectively reduced (because of the dielectric constant) whereas the B field is effectively increased (because of the low effective mass). Thus, even for low-lying impurity and exciton levels in a semiconductor in fields $\approx 10^4 G$, it is possible that magnetic forces dominate. In essence we have a
\[ z_{\text{eff}} = \left( \frac{\nu_{\text{eff}}}{\nu} \right) \] (6)

where \( \nu, \nu_{\text{eff}} \) and \( \varepsilon \) denote the isolated mass, effective mass and dielectric constant respectively. For In Sb we find\(^{14} \) that
\[ z_{\text{eff}} \approx 8.1 \times 10^{-4} \] and that \( B' = B_0 \) for \( B \) as low as \( 1.6 \times 10^3 \) G. Utilizing a Debye-Hückel screening potential, we have carried out\(^{14} \) an accurate multiparameter variational calculation of the energy levels, which is valid for all values of \( B \) and screening length.

REFERENCES
