\[ F(k, E) = \int dt \exp iEt \langle [S_z(t), S_z(0)] \rangle \]  
\[ \lim_{k \to 0} F(k, E) = 0 \]  
and therefore eq. (4) contains no relevant information about gapless modes.

The second flaw in the proof is that even if one works with the proper non-trivial spectral functions pertaining to the \( S^+(k)S^-(\mathbf{k}) \) or \( S^+(k)S^+(\mathbf{k}) \) Green function, and arrives at a formula like

\[ \lim_{k \to 0} E(k) = \lim_{k \to 0} \langle [H, S^+(k, 0)] S^-(\mathbf{-k}, 0) \rangle, \]

and even though this double commutator is zero at \( k = 0 \), a careful analysis [2] shows that such double commutators do not necessarily approach their \( k = 0 \) value in the limit of zero \( k \).

Such limits are only well behaved if the Hamiltonian contains no excessively long range forces [3].

The proof suggested by Crisan is therefore incorrect. The use of Green function notation does not make unnecessary the analysis found in previous proofs of the theorem.

References

MOTION OF A RELATIVISTIC ELECTRON WITH AN ANOMALOUS MAGNETIC MOMENT IN A CONSTANT MAGNETIC FIELD

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The anomalous magnetic moment of the electron is accounted for by adding a phenomenological Pauli term to the Dirac equation. The resultant eigenvalues are applied to (a) the non-linear Lagrangian of the E-M field and (b) spontaneous pair production.

The Dirac equation for an electron of mass \( m \) with an anomalous magnetic moment, \( \mu \) say, in a constant homogeneous magnetic field \( H \) (directed along the \( z \) axis, say) takes the form [1,2] (in our units \( c = \hbar = 1 \) and \( \alpha = e^2 = 1/137 \))

\[ i \frac{\partial \psi}{\partial t} = \left\{ \alpha \cdot (P + eA) + \gamma_4 m + \mu \gamma_4 \Sigma \cdot H \right\} \]  
where the term containing \( \mu \) (the so-called Pauli anomalous interaction term) is an addition to the usual Dirac Hamiltonian. Since different values for the energy eigenvalues derived from this equation appear in the literature [1,2], we have re-derived the result in a conventional manner, and we find, in agreement with ref. 2, that the energy eigenvalues \( E \) are given by

\[ E = \pm \left\{ p_z^2 + \frac{m^2}{H_C} \left( 2n + \xi + 1 \right) \right\}^{1/2} + \frac{\mu H}{2} \]  

where \( n = 0, 1, 2, \ldots \) is the principal quantum number, \( \xi = \pm 1 \) refers to spin up and spin down, \( p_z \) is the momentum of the particle along the \( z \) axis and \( H_C = m^2/e = 4.4 \times 10^{13} \) gauss. For our purposes it is sufficient to take \( \mu = (\alpha/2\pi) \mu_B \) where \( \mu_B \) is the Bohr magneton and so we can write

\[ E = \pm \left\{ p_z^2 + m^2 \left( 1 + \frac{H}{H_C} \left( 2n + \xi + 1 \right) \right)^{1/2} + \frac{\alpha}{4\pi} \frac{H}{H_C} \right\}^{1/2} \]  

Let us now consider applications of eq. (3).

(a) An exact expression for the non-linear Lagrangian of the electromagnetic field has been derived [3,4] in the case where \( \mu = 0 \) (i.e. neglect of the anomalous magnetic moment). Using the more general eigenvalue given by eq. (3) we have...
derived an additional non-linear correction, $L_2$ say, to the Lagrangian of the magnetic field (the electric field will be considered later) as follows ($H^* = H/H_C$)

$$L_2 = \frac{m^2}{32\pi^2} \left( \frac{a}{2\pi} \right)^2 \frac{H^*}{H_C^2} \times$$

$$\times \int_0^{\infty} \frac{d\eta}{\eta^2} \exp(-\eta) \left\{ \eta H^* \coth(\eta H^*) - 1 \right\}$$

In the weak and strong field limits we find

$$L_2 = \frac{m^4}{96\pi^2} \left( \frac{a}{2\pi} \right)^2 H^*^4 \quad \text{for } H^* \ll 1 \quad (5)$$

$$L_2 = \frac{m^4}{32\pi^2} \left( \frac{a}{2\pi} \right)^2 H^*^2 \ln(H/H_C) \quad \text{for } H^* \gg 1 \quad (6)$$

(b) It will be noticed from eq. (3) that, for values $\rho_z = 0$, $n = 0$ and $\xi = -1$, we get a minimum value for $|E|$ given by

$$|E|_{\min} = m \left\{ 1 - \frac{a}{4\pi} H^* \right\}$$

Thus the minimum separation between positive and negative energy states, $\Delta E$ say, is $2m(1 - aH^*/4\pi)$ and so we can conclude that, for values of $H$ equal to $4\pi a^{-1}H_C$, the minimum separation is zero and thus spontaneous pair production may occur. This has important astrophysical implications particularly with respect to the expanding universe if a primordial magnetic field [5] exists. A detailed exposition of the above work will be published elsewhere.

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**References**