Lecture 19: THU 25 MAR 2010

Ch30.5–9
Induction and Inductance II

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Start Your Engines: The Ignition Coil

- The gap between the spark plug in a combustion engine needs an electric field of \( \sim 10^7 \) V/m in order to ignite the air-fuel mixture. For a typical spark plug gap, one needs to generate a potential difference \( > 10^4 \) V!

- But, the typical EMF of a car battery is 12 V. So, how does a spark plug work??

Breaking the circuit changes the current through “primary coil”
- Result: LARGE change in flux thru secondary -- large induced EMF!

The “ignition coil” is a double layer solenoid:
- Primary: small number of turns -- 12 V
- Secondary: MANY turns -- spark plug

http://www.familycar.com/Classroom/ignition.htm
Start Your Engines: The Ignition Coil

The battery establishes a large current in the low-resistance primary coil.

The "points" are opened by cam action to quickly interrupt the current in the primary coil.

Many turns on the secondary coil compared to the primary coil forms a transformer with a large multiplication of voltage.

The capacitor, or "condenser" helps to handle the surge of voltage from the switch action which might otherwise cause sparking across the points.

The sudden change in magnetic field in the primary from the switching off of the current induces a very high voltage in the secondary coil by Faraday's Law.

Transformer: \( P = iV \)
Changing B-Field Produces E-Field!

- We saw that a time varying magnetic flux creates an induced EMF in a wire, exhibited as a current.
- Recall that a current flows in a conductor because of electric field.
- Hence, a time varying magnetic flux must induce an ELECTRIC FIELD!
- A Changing B-Field Produces an E-Field in Empty Space!

\[ \oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt} \]

To decide SIGN of flux, use right hand rule: curl fingers around loop, +flux -- thumb.
Example

• The figure shows two circular regions $R_1$ & $R_2$ with radii $r_1 = 1\text{m}$ & $r_2 = 2\text{m}$. In $R_1$, the magnetic field $B_1$ points out of the page. In $R_2$, the magnetic field $B_2$ points into the page.

• Both fields are uniform and are DECREASING at the SAME steady rate $= 1\ \text{T/s}$.

• Calculate the “Faraday” integral for the two paths shown.

Path I: $\oint C \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt} = -(\pi r_1^2)(-1\ T/\ s) = +3.14V$

Path II:

$$\oint C \vec{E} \cdot d\vec{s} = -\left[ (\pi r_1^2)(-1\ T/\ s) + (\pi r_2^2)(-1\ T/\ s) \right] = +9.42V$$
Solenoid Example

- A long solenoid has a circular cross-section of radius \( R \).
- The magnetic field \( B \) through the solenoid is increasing at a steady rate \( dB/dt \).
- Compute the variation of the electric field as a function of the distance \( r \) from the axis of the solenoid.

First, let’s look at \( r < R \):

\[
|E| (2\pi r) = (\pi r^2) \frac{dB}{dt} \\
E = \frac{r}{2} \frac{dB}{dt}
\]

Next, let’s look at \( r > R \):

\[
|E| (2\pi r) = (\pi R^2) \frac{dB}{dt} \\
E = \frac{R^2}{2r} \frac{dB}{dt}
\]

magnetic field lines

electric field lines
Solenoid Example Cont.

\[ E_{r<r} = \frac{r \ dB}{2 \ dt} \]
\[ \propto r \]

\[ E_{r>r} = \frac{R^2 \ dB}{2r \ dt} \]
\[ \propto 1/r \]

 Added Complication: Changing B Field Is Produced by Changing Current \( i \) in the Loops of Solenoid!

\[ E_{r<r} = \frac{\mu_0 nr \ di}{2} \]
\[ E_{r>r} = \frac{\mu_0 nR^2 \ di}{2r} \]

\[ B = \mu_0 ni \]
\[ \frac{dB}{dt} = \mu_0 n \frac{di}{dt} \]
Summary

Two versions of Faraday's law:

- A time varying magnetic flux produces an EMF:

\[ \mathcal{E} = -\frac{d\Phi_B}{dt} \]

- A time varying magnetic flux produces an electric field:

\[ \oint_{C} \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt} \]
Inductors are with respect to the magnetic field what capacitors are with respect to the electric field. They “pack a lot of field in a small region”. Also, the higher the current, the higher the magnetic field they produce.

**Capacitance** → how much **potential** for a given charge: \( Q = CV \)

**Inductance** → how much **magnetic flux** for a given current: \( \Phi = LI \)

Using Faraday’s law:

\[
\mathcal{E} = -\frac{d\Phi}{dt} = -L \frac{di}{dt}
\]

Units: \([L] = \frac{\text{Tesla} \cdot \text{m}^2}{\text{Ampere}} \equiv \text{H} \) (Henry)

Joseph Henry (1799-1878)
Inductance

Consider a solenoid of length $\ell$ that has $N$ loops of area $A$ each, and $n = \frac{N}{\ell}$ windings per unit length. A current $i$ flows through the solenoid and generates a uniform magnetic field $B = \mu_0 ni$ inside the solenoid. The solenoid magnetic flux is $\Phi_B = NBA$.

$$L = \mu_0 n^2 \ell A$$

The total number of turns $N = n\ell \rightarrow \Phi_B = \left(\mu_0 n^2 \ell A\right)i$. The result we got for the special case of the solenoid is true for any inductor: $\Phi_B = Li$. Here $L$ is a constant known as the inductance of the solenoid. The inductance depends on the geometry of the particular inductor.

Inductance of the Solenoid

For the solenoid, $L = \frac{\Phi_B}{i} = \frac{\mu_0 n^2 \ell Ai}{i} = \mu_0 n^2 \ell A$. 

\[ \]
**Self - Induction**

In the picture to the right we already have seen how a change in the current of loop 1 results in a change in the flux through loop 2, and thus creates an induced emf in loop 2.

If we change the current through an inductor this causes a change in the magnetic flux $\Phi_B = Li$ through the inductor according to the equation $\frac{d\Phi_B}{dt} = L\frac{di}{dt}$. Using Faraday's law we can determine the resulting emf known as self-induced emf: $E = -\frac{d\Phi_B}{dt} = -L\frac{di}{dt}$.

**SI unit for** $L$: the henry (symbol: H)

An inductor has inductance $L = 1 \text{ H}$ if a current change of 1 A/s results in a self-induced emf of 1 V.
\[ \mathcal{E} = -L \frac{di}{dt} \]

Example

- The current in a L=10H inductor is decreasing at a steady rate of \( i = 5 \text{A/s} \).
- If the current is as shown at some instant in time, what is the magnitude and direction of the induced EMF?

\[ \mathcal{E} = (10 \text{H})(5 \text{A/s}) = 50 \text{V} \]

- Current is decreasing
- Induced EMF must be in a direction that OPPOSES this change.
- So, induced EMF must be in same direction as current

(a) 50 V

(b) 50 V
The RL circuit

- Set up a single loop series circuit with a battery, a resistor, a solenoid and a switch.
- Describe what happens when the switch is closed.
- Key processes to understand:
  - What happens JUST AFTER the switch is closed?
  - What happens a LONG TIME after switch has been closed?
  - What happens in between?

Key insights:
- You cannot change the CURRENT in an inductor instantaneously!
- If you wait long enough, the current in an RL circuit stops changing!

At $t = 0$, a capacitor acts like a wire; an inductor acts like a broken wire.
At $t = \infty$ a capacitor acts like a broken wire, and inductor acts like a short circuit.
In an RC circuit, while charging, $Q = CV$ and the loop rule mean:

- charge increases from $0$ to $CE$
- current decreases from $\frac{\mathcal{E}}{R}$ to $0$
- voltage across capacitor increases from $0$ to $\mathcal{E}$

In an RL circuit, while “charging” (rising current), $\mathcal{E} = L\frac{di}{dt}$ and the loop rule mean:

- magnetic field increases from $0$ to $B$
- current increases from $0$ to $\frac{\mathcal{E}}{R}$
- voltage across inductor decreases from $-\mathcal{E}$ to $0$
Immediately after the switch is closed, what is the potential difference across the inductor?

(a) 0 V
(b) 9 V
(c) 0.9 V

- Immediately after the switch, current in circuit = 0.
- So, potential difference across the resistor = 0!
- So, the potential difference across the inductor = $\mathcal{E} = 9 \text{ V}$!
Example

• **Immediately after the switch is closed,** what is the current $i$ through the 10 Ω resistor?
  - (a) 0.375 A
  - (b) 0.3 A
  - (c) 0

  **(c) 0**

• **Long after the switch has been closed,** what is the current in the 40 Ω resistor?
  - (a) 0.375 A
  - (b) 0.3 A
  - (c) 0.075 A

  **(c) 0.075 A**

**Immediate after switch is closed**, current through inductor = 0.
Hence, current through battery and through 10 Ω resistor is
\[ i = \frac{3 \text{ V}}{10 \Omega} = 0.3 \text{ A} \]

**Long after switch is closed**, potential across inductor = 0.
Hence, current through 40 Ω resistor
\[ i = \frac{3 \text{ V}}{40 \Omega} = 0.075 \text{ A (Par-V)} \]
Fluxing Up The Inductor

• How does the current in the circuit change with time?

\[-iR + \mathcal{E} - L \frac{di}{dt} = 0\]

\[i = \frac{\mathcal{E}}{R} \left(1 - e^{-t/\tau}\right)\]

Time constant of RL circuit: \(\tau = L/R\)
RL Circuit Movie
Fluxing Down an Inductor

The switch is at $a$ for a long time, until the inductor is charged. Then, the switch is closed to $b$.

What is the current in the circuit?

Loop rule around the new circuit:

$$iR + L \frac{di}{dt} = 0$$

$$i(t) = \frac{\mathcal{E}}{R} e^{-Rt/L} = \frac{\mathcal{E}}{R} e^{-t/\tau}$$

$$\tau_{RL} = \frac{L}{R}$$
Inductors & Energy

- Recall that **capacitors** store energy in an **electric** field.
- **Inductors** store energy in a **magnetic** field.

\[ E = iR + L \frac{di}{dt} \]

\[ (iE) = (i^2R) + Li \frac{di}{dt} \]

\[ (iE) = (i^2R) + \frac{d}{dt} \left( \frac{Li^2}{2} \right) \]

\[ P = iV = i^2R \]

Power delivered by battery = power dissipated by R

+ (d/dt) energy stored in L
Inductors & Energy

\[ U_B = \frac{Li^2}{2} \]

Magnetic Potential Energy \( U_B \) Stored in an Inductor.

\[ P = Li \frac{di}{dt} \]

Magnetic Power Returned from Defluxing Inductor to Circuit.
Example

• The switch has been in position “a” for a long time.
• It is now moved to position “b” without breaking the circuit.
• What is the total energy dissipated by the resistor until the circuit reaches equilibrium?

When switch has been in position “a” for long time, current through inductor = \( \frac{9\text{V}}{10\Omega} \) = 0.9A.
Energy stored in inductor = \( 0.5(10\text{H})(0.9\text{A})^2 \) = 4.05 J
When inductor “discharges” through the resistor, all this stored energy is dissipated as heat = 4.05 J.
$E=120\text{V}$, $R_1=10\Omega$, $R_2=20\Omega$, $R_3=30\Omega$, $L=3\text{H}$.

1. What are $i_1$ and $i_2$ immediately after closing the switch?
2. What are $i_1$ and $i_2$ a long time after closing the switch?
3. What are $i_1$ and $i_2$ immediately after reopening the switch?
4. What are $i_1$ and $i_2$ a long time after reopening the switch?