Lecture 12: TUE 02 FEB

DC circuits
Ch27.4–9

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How to Solve Multi-Loop Circuits
Step I: Simplify “Compile” Circuits

Resistors

Key formula: \( V = iR \)

In series: same current \( \frac{dQ}{dt} \)

\[ R_{eq} = \sum R_j \]

In parallel: same voltage

\[ \frac{1}{R_{eq}} = \sum \frac{1}{R_j} \]

\[ P = iV = i^2R = V^2/R \]

Capacitors

\( Q = CV \)

In parallel: same voltage

\[ \frac{1}{C_{eq}} = \sum \frac{1}{C_j} \]

\[ U = \frac{QV}{2} = \frac{Q^2}{2C} = CV^2 \]
Note: Skip Compile and Go Straight to Loop and Junction Rules if Number of Batteries is Large

One Battery: Compile First

Three Batteries: Straight to Loop & Junction
Step II: Apply Loop Rule

Around every loop add $+\mathcal{E}$ if you cross a battery from minus to plus, $-\mathcal{E}$ if plus to minus, and $-iR$ for each resistor. Then sum to Zero: $+\mathcal{E}_1 - \mathcal{E}_2 - iR_1 - iR_2 = 0$.

Conservation of ENERGY!
Step II: Apply Junction Rule

At every junction sum the ingoing currents and outgoing currents and set them equal.

\[ i_1 = i_2 + i_3 \]

Conservation of CHARGE!
Step III: Equations to Unknowns

Continue Steps I–III until you have as many equations as unknowns!

Given: \( E_1, E_2, i, R_1, R_2 \)

\[
+ E_1 - E_2 - i_1 R_1 - i_2 R_2 = 0
\]

and

\[
i = i_1 + i_2
\]

Solve for \( i_2, i_3 \)
Example

Find the equivalent resistance between points
(a) $F$ and $H$ and
(b) $F$ and $G$.

(Hint: For each pair of points, imagine that a battery is connected across the pair.)
Assume the batteries are ideal, and have emf $E_1=8\text{V}$, $E_2=5\text{V}$, $E_3=4\text{V}$, and $R_1=140\Omega$, $R_2=75\Omega$ and $R_3=2\Omega$.

What is the current in each branch?
What is the power delivered by each battery?
Which point is at a higher potential, a or b?

Apply loop rule three times and junction rule rule twice.
• What’s the current through resistor $R_1$?
• What’s the current through resistor $R_2$?
• What’s the current through each battery?

Apply loop rule three times and junction rule twice.
Non-Ideal Batteries

- You have two ideal identical batteries, and a resistor. Do you connect the batteries in series or in parallel to get maximum current through $R$?
- Does the answer change if you have non-ideal (but still identical) batteries?

Apply loop and junction rules until you have current in $R$. 

(a) 

(b)
More Light Bulbs

• If all batteries are ideal, and all batteries and light bulbs are identical, in which arrangements will the light bulbs as bright as the one in circuit X?

• Does the answer change if batteries are not ideal?

Calculate $i$ and $V$ across each bulb. $P = iV = \text{“brightness”}$
or
Calculate each $i$ with $R$’s the same: $P = i^2R$
**RC Circuits: Charging a Capacitor**

In these circuits, current will change for a while, and then stay constant. We want to solve for current as a function of time \( i(t) = dq/dt \).

The charge on the capacitor will also be a function of time: \( q(t) \).

The voltage across the resistor and the capacitor also change with time.

To charge the capacitor, close the switch on \( a \).

\[ V_C = Q/C \]
\[ V_R = iR \]

A differential equation for \( q(t) \)!

The solution is:

\[
q(t) = C \mathcal{E} \left( 1 - e^{-t/RC} \right)
\]

\[ \rightarrow \quad i(t) \equiv dq/dt = \left( \mathcal{E} / R \right) e^{-t/RC} \]

Time constant: \( \tau = RC \)
RC Circuits: Discharging a Capacitor

Assume the switch has been closed on \( a \) for a long time: the capacitor will be charged with \( Q = CE \).

Then, close the switch on \( b \): charges find their way across the circuit, establishing a current.

\[
V_R + V_C = 0 \\
-i(t)R + \frac{q(t)}{C} = 0 \implies (dq/dt)R + \frac{q(t)}{C} = 0
\]

Solution:
\[
q(t) = q(0)e^{-t/RC} = CEe^{-t/RC} \\
i(t) = \frac{dq}{dt} = \left(\frac{q(0)}{RC}\right)e^{-t/RC} = \left(\frac{E}{R}\right)e^{-t/RC}
\]
Example

The three circuits below are connected to the same ideal battery with emf $E$. All resistors have resistance $R$, and all capacitors have capacitance $C$.

- Which capacitor takes the longest in getting charged?
- Which capacitor ends up with the largest charge?
- What’s the final current delivered by each battery?
- What happens when we disconnect the battery?

Compile $R$’s into $R_{eq}$. Then apply charging formula with $R_{eq}C = \tau$. 

(1) \hspace{2cm} (2) \hspace{2cm} (3)
Example

In the figure, $E = 1$ kV, $C = 10$ µF, $R_1 = R_2 = R_3 = 1$ MΩ. With $C$ completely uncharged, switch $S$ is suddenly closed (at $t = 0$).

- What’s the current through each resistor at t=0?
- What’s the current through each resistor after a long time?
- How long is a long time?

Compile $R_1$, $R_2$, and $R_3$ into $R_{eq}$. Then apply discharging formula with $R_{eq}C = \tau$. 