Physics 2102
Lecture: 08 THU 18 FEB
Capacitance II

25.4–7

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Capacitors in Parallel: \( V = \text{Constant} \)

- An ISOLATED wire is an equipotential surface: \( V = \text{Constant} \)
- Capacitors in parallel have SAME potential difference but NOT ALWAYS same charge!
- \( V_{AB} = V_{CD} = V \)
- \( Q_{\text{total}} = Q_1 + Q_2 \)
- \( C_{eq} \cdot V = C_1 \cdot V + C_2 \cdot V \)
- \( C_{eq} = C_1 + C_2 \)
- Equivalent \text{parallel} capacitance = sum of capacitances

PAR-V (Parallel: \( V \) the Same)
Capacitors in Series: $Q = \text{Constant}$

- $Q_1 = Q_2 = Q = \text{Constant}$
- $V_{AC} = V_{AB} + V_{BC}$

SERI-Q: Series $Q$ the Same

\[
\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2}
\]

\[
\frac{1}{C_{\text{series}}} = \frac{1}{C_1} + \frac{1}{C_2}
\]

SERIES:
- $Q$ is same for all capacitors
- Total potential difference = sum of $V$

Isolated Wire: $Q = Q_1 = Q_2 = \text{Constant}$

$Q = \frac{Q_1}{C_{eq}} = \frac{Q_2}{C_{eq}}$
Capacitors in Parallel and in Series

- **In parallel:**
  \[ C_{\text{par}} = C_1 + C_2 \]
  \[ V_{\text{par}} = V_1 = V_2 \]
  \[ Q_{\text{par}} = Q_1 + Q_2 \]

- **In series:**
  \[ \frac{1}{C_{\text{ser}}} = \frac{1}{C_1} + \frac{1}{C_2} \]
  \[ V_{\text{ser}} = V_1 + V_2 \]
  \[ Q_{\text{ser}} = Q_1 = Q_2 \]
What is the charge on each capacitor?

- \( Q_i = C_i V \)
- \( V = 120\text{V} \) on ALL Capacitors (PAR-V)
- \( Q_1 = (10 \ \mu\text{F})(120\text{V}) = 1200 \ \mu\text{C} \)
- \( Q_2 = (20 \ \mu\text{F})(120\text{V}) = 2400 \ \mu\text{C} \)
- \( Q_3 = (30 \ \mu\text{F})(120\text{V}) = 3600 \ \mu\text{C} \)

Note that:
- Total charge (7200 \( \mu\text{C} \)) is shared between the 3 capacitors in the ratio \( C_1:C_2:C_3 = \) i.e. 1:2:3

\[
C_{\text{par}} = C_1 + C_2 + C_3 = (10 + 20 + 30)\mu\text{F} = 60\mu\text{F}
\]
Example: Parallel or Series

Series: Isolated Islands (Constant Q)

What is the potential difference across each capacitor?

- \( Q = C_{ser}V \)
- \( Q \) is same for all capacitors (SERI-Q)
- Combined \( C_{ser} \) is given by:

\[
\frac{1}{C_{ser}} = \frac{1}{(10 \mu F)} + \frac{1}{(20 \mu F)} + \frac{1}{(30 \mu F)}
\]

- \( C_{eq} = 5.46 \mu F \) (solve above equation)
- \( V_1 = \frac{Q}{C_1} = \frac{655 \mu C}{(10 \mu F)} = 65.5 \) V
- \( V_2 = \frac{Q}{C_2} = \frac{655 \mu C}{(20 \mu F)} = 32.75 \) V
- \( V_3 = \frac{Q}{C_3} = \frac{655 \mu C}{(30 \mu F)} = 21.8 \) V

Note: 120V is shared in the ratio of INVERSE capacitances i.e. 
(1):(1/2):(1/3) 
(largest C gets smallest V)
Example: Series or Parallel?

Neither: Circuit Compilation Needed!

In the circuit shown, what is the charge on the 10\( \mu \)F capacitor?

- The two 5\( \mu \)F capacitors are in parallel.
- Replace by 10\( \mu \)F.
- Then, we have two 10\( \mu \)F capacitors in series.
- So, there is 5V across the 10 \( \mu \)F capacitor of interest by symmetry.
- Hence, \( Q = (10\mu F)(5V) = 50\mu C \).
Energy $U$ Stored in a Capacitor

- Start out with uncharged capacitor
- Transfer small amount of charge $dq$ from one plate to the other until charge on each plate has magnitude $Q$
- How much work was needed?

\[
U = \int_{0}^{Q} Vdq = \int_{0}^{Q} \frac{q}{C} dq = \frac{Q^2}{2C} = \frac{CV^2}{2}
\]
Energy Stored in Electric Field of Capacitor

- Energy stored in capacitor: \( U = \frac{Q^2}{2C} = CV^2/2 \)
- View the energy as stored in ELECTRIC FIELD
- For example, parallel plate capacitor: Energy DENSITY = energy/volume = \( u = \)

\[
u = \frac{Q^2}{2CAd} = \frac{Q^2}{2\varepsilon_0 A^2} \left( \frac{\varepsilon_0 A}{d} \right) = \frac{Q^2}{2\varepsilon_0 A^2} \left( \frac{Q}{\varepsilon_0 A} \right)^2 = \frac{\varepsilon_0 E^2}{2}
\]

Volume = Ad

[General expression for any region with vacuum (or air)]
Dielectric Constant

- If the space between capacitor plates is filled by a dielectric, the capacitance INCREASES by a factor $\kappa$.
- This is a useful, working definition for dielectric constant.
- Typical values of $\kappa$ are 10–200.

The $\kappa$ and the constant $\varepsilon = \kappa \varepsilon_0$ are both called dielectric constants. The $\kappa$ has no units (dimensionless).

$C = \kappa \varepsilon_0 A/d$
Molecules set up counter $E_{mol}$ field that somewhat cancels out capacitor field $E_{cap}$.

This avoids sparking (dielectric breakdown) by keeping field inside dielectric small.

Hence the bigger the dielectric constant the more charge you can store on the capacitor.
• Capacitor has charge $Q$, voltage $V$
• Battery remains connected while dielectric slab is inserted.
• Do the following increase, decrease or stay the same:
  - Potential difference?
  - Capacitance?
  - Charge?
  - Electric field?
Example

• Initial values:
capacitance = \( C \); charge = \( Q \); potential
difference = \( V \); electric field = \( E \);

• Battery remains connected
• \( V \) is FIXED; \( V_{\text{new}} = V \) (same)
• \( C_{\text{new}} = \kappa C \) (increases)
• \( Q_{\text{new}} = (\kappa C)V = \kappa Q \) (increases).
• Since \( V_{\text{new}} = V \), \( E_{\text{new}} = V/d = E \) (same)

Energy stored? \( u = \varepsilon_0 E^2/2 \implies u = \kappa \varepsilon_0 E^2/2 = \varepsilon E^2/2 \)
Summary

- Any two charged conductors form a capacitor.
- Capacitance: \( C = \frac{Q}{V} \)

- Simple Capacitors:
  
  - Parallel plates: \( C = \varepsilon_0 \frac{A}{d} \)
  
  - Spherical: \( C = 4\pi \varepsilon_0 \frac{ab}{(b-a)} \)
  
  - Cylindrical: \( C = 2\pi \varepsilon_0 \frac{L}{\ln(b/a)} \)

- Capacitors in series: same charge, not necessarily equal potential; equivalent capacitance \( \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots \)

- Capacitors in parallel: same potential; not necessarily same charge; equivalent capacitance \( C_{eq} = C_1 + C_2 + \ldots \)

- Energy in a capacitor: \( U = \frac{Q^2}{2C} = CV^2/2 \); energy density \( u = \varepsilon_0 E^2/2 \)

- Capacitor with a dielectric: capacitance increases \( C' = kC \)