Physics 2102
Lecture: 08 THU 11 FEB
Capacitance I

25.1–4
Capacitors and Capacitance

Capacitor: any two conductors, one with charge $+Q$, other with charge $-Q$

Potential DIFFERENCE between conductors = $V$

$Q = CV$ where $C = \text{capacitance}$

Units of capacitance: Farad (F) = Coulomb/Volt

Uses: storing and releasing electric charge/energy.
Most electronic capacitors:
- micro-Farads ($\mu$F),
- pico-Farads (pF) — $10^{-12}$ F
New technology: compact 1 F capacitors
Capacitance

- Capacitance depends only on GEOMETRICAL factors and on the MATERIAL that separates the two conductors.
- e.g. Area of conductors, separation, whether the space in between is filled with air, plastic, etc.

(We first focus on capacitors where gap is filled by AIR!)
Electrolytic (1940-70)  Electrolytic (new)

Paper (1940-70)

Tantalum (1980 on)

Ceramic (1930 on)

Mica (1930-50)
Parallel Plate Capacitor

We want capacitance: \( C = \frac{Q}{V} \)

E field between the plates: (Gauss' Law)

\[
E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}
\]

Relate \( E \) to potential difference \( V \):

\[
V = \int_{0}^{d} \vec{E} \cdot d\vec{x} = \int_{0}^{d} \frac{Q}{\varepsilon_0 A} \, dx = \frac{Qd}{\varepsilon_0 A}
\]

What is the capacitance \( C \)?

\[
C = \frac{Q}{V} = \frac{\varepsilon_0 A}{d}
\]

Units:

\[
\frac{C^2}{Nm^2 \cdot m} = \frac{C^2}{Nm} = \frac{CC}{J} = \frac{C}{V}
\]
Capacitance and Your iPhone!

\[ C = \frac{Q}{V} = \frac{\varepsilon_0 A}{d} \]

1. A capacitive sensor is a solid-state sensor made using standard pc-board or flex circuit technology. A finger on top of a grid of conductive traces changes the capacitance of the nearest traces. This change in trace capacitance can be measured, and finger position can be computed.
Parallel Plate Capacitor — Example

• A huge parallel plate capacitor consists of two square metal plates of side 50 cm, separated by an air gap of 1 mm
• What is the capacitance?

\[ C = \varepsilon_0 \frac{A}{d} \]

\[ = (8.85 \times 10^{-12} \text{ F/m})(0.25 \text{ m}^2)/(0.001 \text{ m}) \]

\[ = 2.21 \times 10^{-9} \text{ F} \]

(Very Small!!)

Lesson: difficult to get large values of capacitance without special tricks!

Units:

\[ \left[ \frac{C^2}{\text{Nm}^2 \text{m}} \right] = \left[ \frac{C^2}{\text{Nm}} \right] = \left[ \frac{CC}{\text{J}} \right] = \left[ \frac{C}{\text{V}} \right] \equiv [\text{F}] = \text{Farad} \]
Isolated Parallel Plate Capacitor

- A parallel plate capacitor of capacitance $C$ is charged using a battery.
- Charge = $Q$, potential difference = $V$.
- Battery is then disconnected.
- If the plate separation is INCREASED, does Potential Difference $V$:
  
(a) Increase? ★
(b) Remain the same?
(c) Decrease?

- $Q$ is fixed!
- $C$ decreases ($=\varepsilon_0 A/d$)
- $V=Q/C$; $V$ increases.
A parallel plate capacitor of capacitance $C$ is charged using a battery.

Charge = $Q$, potential difference = $V$.

Plate separation is INCREASED while battery remains connected.

Does the Electric Field Inside:
(a) Increase?
(b) Remain the Same?
(c) Decrease?

- $V$ is fixed by battery!
- $C$ decreases ($=\varepsilon_0 A/d$)
- $Q=CV$; $Q$ decreases
- $E = Q/\varepsilon_0 A$ decreases
Spherical Capacitor

What is the electric field inside the capacitor? (Gauss’ Law)

\[ E = \frac{Q}{4\pi\varepsilon_0 r^2} \]

Relate \( E \) to potential difference between the plates:

\[ V = \int_{a}^{b} \vec{E} \cdot d\vec{r} = \int_{a}^{b} \frac{kQ}{r^2} dr = \left[ -\frac{kQ}{r} \right]_{a}^{b} = kQ \left[ \frac{1}{a} - \frac{1}{b} \right] \]

Radius of outer plate = \( b \)
Radius of inner plate = \( a \)

Concentric spherical shells:
Charge +\( Q \) on inner shell, -\( Q \) on outer shell
What is the capacitance?

\[ C = \frac{Q}{V} = \]

\[ \frac{Q}{4\pi\varepsilon_0} \left[ \frac{1}{a} - \frac{1}{b} \right] \]

= \frac{4\pi\varepsilon_0 ab}{(b - a)}

Concentric spherical shells:
Charge \( +Q \) on inner shell,
\(-Q \) on outer shell

Isolated sphere: let \( b \gg a \),

\[ C = 4\pi\varepsilon_0 a \]

Radius of outer plate = \( b \)
Radius of inner plate = \( a \)
Cylindrical Capacitor

What is the electric field in between the plates? Gauss’ Law!

\[ E = \frac{Q}{2\pi \varepsilon_0 rL} \]

Relate \( E \) to potential difference between the plates:

\[ V = \int_{a}^{b} \vec{E} \cdot d\vec{r} \]

\[ = \int_{a}^{b} \frac{Q}{2\pi \varepsilon_0 rL} dr = \left[ \frac{Q \ln r}{2\pi \varepsilon_0 L} \right]_{a}^{b} \]

\[ = \frac{Q}{2\pi \varepsilon_0 L} \ln \left( \frac{b}{a} \right) \]

Cylindrical Gaussian surface of radius \( r \)

Radius of outer plate = \( b \)
Radius of inner plate = \( a \)
Length of capacitor = \( L \)
+\( Q \) on inner rod, -\( Q \) on outer shell
• Any two charged conductors form a capacitor.

• Capacitance: \( C = \frac{Q}{V} \)

• Simple Capacitors:

  - Parallel plates: \( C = \varepsilon_0 \frac{A}{d} \)
  - Spherical: \( C = 4\pi \varepsilon_0 \frac{ab}{(b-a)} \)
  - Cylindrical: \( C = 2\pi \varepsilon_0 \frac{L}{\ln(b/a)} \)
Capacitors in Parallel: V=Constant

- An ISOLATED wire is an equipotential surface: V=Constant
- Capacitors in parallel have SAME potential difference but NOT ALWAYS same charge!
- $V_{AB} = V_{CD} = V$
- $Q_{total} = Q_1 + Q_2$
- $C_{eq}V = C_1V + C_2V$
- $C_{eq} = C_1 + C_2$
- Equivalent parallel capacitance = sum of capacitances

$C_{parallel} = C_1 + C_2$

PAR-V (Parallel V the Same)
Capacitors in Series: $Q = \text{Constant}$

- $Q_1 = Q_2 = Q = \text{Constant}$
- $V_{AC} = V_{AB} + V_{BC}$

SERI-Q (Series Q the Same)

\[
\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2}
\]

\[
\frac{1}{C_{\text{series}}} = \frac{1}{C_1} + \frac{1}{C_2}
\]

SERIES:
- $Q$ is same for all capacitors
- Total potential difference = sum of $V$

Isolated Wire:
$Q = Q_1 = Q_2 = \text{Constant}$
Capacitors in parallel and in series

- **In parallel:**
  - \( C_{\text{par}} = C_1 + C_2 \)
  - \( V_{\text{par}} = V_1 = V_2 \)
  - \( Q_{\text{par}} = Q_1 + Q_2 \)

- **In series:**
  - \( 1/C_{\text{ser}} = 1/C_1 + 1/C_2 \)
  - \( V_{\text{ser}} = V_1 + V_2 \)
  - \( Q_{\text{ser}} = Q_1 = Q_2 \)
Example: Parallel or Series?

Parallel: Circuit Splits Cleanly in Two (Constant V)

What is the charge on each capacitor?

- \( Q_i = C_i V \)
- \( V = 120V = \text{Constant} \)
- \( Q_1 = (10 \ \mu F)(120V) = 1200 \ \mu C \)
- \( Q_2 = (20 \ \mu F)(120V) = 2400 \ \mu C \)
- \( Q_3 = (30 \ \mu F)(120V) = 3600 \ \mu C \)

Note that:
- Total charge (7200 \( \mu C \)) is shared between the 3 capacitors in the ratio \( C_1 : C_2 : C_3 \) — i.e. 1:2:3

\[ C_{par} = C_1 + C_2 + C_3 = (10 + 20 + 30) \mu F = 60 \mu F \]
Example: Parallel or Series

Series: Isolated Islands (Constant Q)

What is the potential difference across each capacitor?

- $Q = C_{ser}V$
- $Q$ is same for all capacitors
- Combined $C_{ser}$ is given by:

\[
\frac{1}{C_{ser}} = \frac{1}{(10 \mu F)} + \frac{1}{(20 \mu F)} + \frac{1}{(30 \mu F)}
\]

- $C_{eq} = 5.46 \mu F$ (solve above equation)
- $Q = C_{eq}V = (5.46 \mu F)(120V) = 655 \mu C$
- $V_1 = \frac{Q}{C_1} = (655 \mu C)/(10 \mu F) = 65.5 \text{ V}$
- $V_2 = \frac{Q}{C_2} = (655 \mu C)/(20 \mu F) = 32.75 \text{ V}$
- $V_3 = \frac{Q}{C_3} = (655 \mu C)/(30 \mu F) = 21.8 \text{ V}$

Note: 120V is shared in the ratio of INVERSE capacitances i.e. (1):(1/2):(1/3) (largest C gets smallest V)
Example: Series or Parallel?

In the circuit shown, what is the charge on the 10\( \mu \)F capacitor?

• The two 5\( \mu \)F capacitors are in parallel
• Replace by 10\( \mu \)F
• Then, we have two 10\( \mu \)F capacitors in series
• So, there is 5V across the 10 \( \mu \)F capacitor of interest
• Hence, \( Q = (10\mu F \times 5V) = 50\mu C \)