Physics 2102
Lecture 8
Capacitors II
Capacitors in parallel and in series

- **In parallel:**
  - $C_{eq} = C_1 + C_2$
  - $V_{eq} = V_1 = V_2$
  - $Q_{eq} = Q_1 + Q_2$

- **In series:**
  - $1/C_{eq} = 1/C_1 + 1/C_2$
  - $V_{eq} = V_1 + V_2$
  - $Q_{eq} = Q_1 = Q_2$
Example 1

What is the charge on each capacitor?

- $Q = CV$; $V = 120$ V
- $Q_1 = (10 \ \mu\text{F})(120\text{V}) = 1200 \ \mu\text{C}$
- $Q_2 = (20 \ \mu\text{F})(120\text{V}) = 2400 \ \mu\text{C}$
- $Q_3 = (30 \ \mu\text{F})(120\text{V}) = 3600 \ \mu\text{C}$

Note that:
- Total charge (7200 \ \mu\text{C}) is shared between the 3 capacitors in the ratio $C_1:C_2:C_3$ — i.e. 1:2:3
Example 2

What is the potential difference across each capacitor?

- $Q = CV$; $Q$ is same for all capacitors
- Combined $C$ is given by:
  \[
  \frac{1}{C_{eq}} = \frac{1}{(10 \mu F)} + \frac{1}{(20 \mu F)} + \frac{1}{(30 \mu F)}
  \]

- $C_{eq} = 5.46 \mu F$
- $Q = CV = (5.46 \mu F)(120V) = 655 \mu C$
- $V_1 = Q/C_1 = (655 \mu C)/(10 \mu F) = 65.5 V$
- $V_2 = Q/C_2 = (655 \mu C)/(20 \mu F) = 32.75 V$
- $V_3 = Q/C_3 = (655 \mu C)/(30 \mu F) = 21.8 V$

Note: 120V is shared in the ratio of INVERSE capacitances
i.e. $1:(1/2):(1/3)$ (largest $C$ gets smallest $V$)
Example 3

In the circuit shown, what is the charge on the 10\( \mu \)F capacitor?

- The two 5\( \mu \)F capacitors are in parallel
- Replace by 10\( \mu \)F
- Then, we have two 10\( \mu \)F capacitors in series
- So, there is 5V across the 10\( \mu \)F capacitor of interest
- Hence, \( Q = (10\mu F)(5V) = 50\mu C \)
Energy Stored in a Capacitor

- Start out with uncharged capacitor
- Transfer small amount of charge $dq$ from one plate to the other until charge on each plate has magnitude $Q$
- How much work was needed?

$$U = \int_{0}^{Q} V dq = \int_{0}^{Q} \frac{q}{C} dq = \frac{Q^2}{2C} = \frac{CV^2}{2}$$
Energy Stored in Electric Field

- Energy stored in capacitor: \( U = \frac{Q^2}{2C} = CV^2/2 \)
- View the energy as stored in ELECTRIC FIELD
- For example, parallel plate capacitor:

\[
U = \frac{Q^2}{2CAd} = \frac{Q^2}{2\varepsilon_0 A^2} = \frac{\varepsilon_0}{2} \left( \frac{Q}{\varepsilon_0 A} \right)^2 = \frac{\varepsilon_0 E^2}{2}
\]

volume = \( Ad \)

Energy DENSITY = energy/volume = \( u = \)  

General expression for any region with vacuum (or air)
**Example**

- 10μF capacitor is initially charged to 120V. 20μF capacitor is initially uncharged.
- Switch is closed, equilibrium is reached.
- How much energy is dissipated in the process?

Initial charge on 10μF = (10μF)(120V) = 1200μC

After switch is closed, let charges = $Q_1$ and $Q_2$.

Charge is conserved: $Q_1 + Q_2 = 1200μC$

Also, $V_{\text{final}}$ is same: $\frac{Q_1}{C_1} = \frac{Q_2}{C_2}$

- $Q_1 = 400μC$
- $Q_2 = 800μC$
- $V_{\text{final}} = \frac{Q_1}{C_1} = 40$ V

Initial energy stored = $(1/2)C_1V_{\text{initial}}^2 = (0.5)(10μF)(120)^2 = 72mJ$

Final energy stored = $(1/2)(C_1 + C_2)V_{\text{final}}^2 = (0.5)(30μF)(40)^2 = 24mJ$

Energy lost (dissipated) = 48mJ
Dielectric Constant

If the space between capacitor plates is filled by a dielectric, the capacitance INCREASES by a factor $\kappa$.

This is a useful, working definition for dielectric constant.

Typical values of $\kappa$ are 10–200.

$$C = \kappa \varepsilon_0 A/d$$

$DIELECTRIC$
Example

- Capacitor has charge $Q$, voltage $V$
- Battery remains connected while dielectric slab is inserted.
- Do the following increase, decrease or stay the same:
  - Potential difference?
  - Capacitance?
  - Charge?
  - Electric field?
Example

• Initial values:
capacitance = \( C \); charge = \( Q \);
potential difference = \( V \);
electric field = \( E \);
• Battery remains connected
• \( V \) is FIXED; \( V_{\text{new}} = V \) (same)
• \( C_{\text{new}} = \kappa C \) (increases)
• \( Q_{\text{new}} = (\kappa C) V = \kappa Q \) (increases).
• Since \( V_{\text{new}} = V \), \( E_{\text{new}} = E \) (same)

Energy stored? \[ u = \varepsilon_0 E^2 / 2 \implies u = \kappa \varepsilon_0 E^2 / 2 = \varepsilon E^2 / 2 \]
Summary

• Any two charged conductors form a capacitor.
• Capacitance : \( C = \frac{Q}{V} \)

• Simple Capacitors:
  - *Parallel plates*: \( C = \varepsilon_0 \frac{A}{d} \)
  - *Spherical*: \( C = 4\pi \varepsilon_0 \frac{ab}{(b-a)} \)
  - *Cylindrical*: \( C = 2\pi \varepsilon_0 \frac{L}{\ln(b/a)} \)

• Capacitors in series: same charge, not necessarily equal potential; equivalent capacitance \( \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots \)
• Capacitors in parallel: same potential; not necessarily same charge; equivalent capacitance \( C_{eq} = C_1 + C_2 + \ldots \)

• Energy in a capacitor: \( U = \frac{Q^2}{2C} = CV^2/2 \); energy density \( u = \varepsilon_0 E^2/2 \)

• Capacitor with a dielectric: capacitance increases \( C' = \kappa C \)