Electric Potential I

Physics 2102
Lecture 5
Electric Potential I
Electric potential energy

Electric potential energy of a system is equal to **minus** the work done by electrostatic forces when building the system (assuming charges were initially infinitely separated)

\[ U = -W_{\infty} \]

The change in potential energy between an initial and final configuration is equal to **minus** the work done by the electrostatic forces:

\[ \Delta U = U_f - U_i = -W \]

- What is the potential energy of a single charge?
- What is the potential energy of a dipole?
- A proton moves from point \( i \) to point \( f \) in a uniform electric field, as shown.
  - Does the electric field do positive or negative work on the proton?
  - Does the electric potential energy of the proton increase or decrease?
**Electric potential**

Electric potential difference between two points = work per unit charge needed to move a charge between the two points:

\[ \Delta V = V_f - V_i = -\frac{W}{q} = \Delta U/q \]

\[ dW = \vec{F} \cdot d\vec{s} \]

\[ dW = q_0 \vec{E} \cdot d\vec{s} \]

\[ W = \int_{i}^{f} dW = \int_{i}^{f} q_0 \vec{E} \cdot d\vec{s} \]

\[ \Delta V = V_f - V_i = -\frac{W}{q_0} = -\int_{i}^{f} \vec{E} \cdot d\vec{s} \]
Electric potential energy, electric potential

Units: \([U] = [W] = \text{Joules};\)
\([V] = [W/q] = \text{Joules/C} = \text{Nm/C} = \text{Volts}\)
\([E] = \text{N/C} = \text{Vm}\)

1 \text{eV} = \text{work needed to move an electron through a potential difference of 1V}: \[W = q \Delta V = e \times 1 \text{V}\]

\[= 1.60 \times 10^{-19} \text{C} \times 1 \text{J/C} = 1.60 \times 10^{-19} \text{J}\]
Equipotential surfaces

\[ \Delta V = V_f - V_i = -\frac{W}{q_0} = -\int_{i}^{f} \vec{E} \cdot d\vec{s} \]

Given a charged system, we can:
• draw electric field lines: the electric field is tangent to the field lines
• draw equipotential surfaces: the electric potential is constant on the surface

• Equipotential surfaces are perpendicular to electric field lines. Why??
• No work is needed to move a charge along an equipotential surface. Why??
• Electric field lines always point towards equipotential surfaces with lower potential. Why??
Electric field lines and equipotential surfaces

http://www.cco.caltech.edu/~phys1/java/phys1/EField/EField.html
Electric potential and electric potential energy

The change in potential energy of a charge $q$ moving from point $i$ to point $f$ is equal to the work done by the applied force, which is equal to minus the work done by the electric field, which is related to the difference in electric potential:

$$\Delta U = U_f - U_i = W_{app} = -W = q\Delta V$$

We move a proton from point $i$ to point $f$ in a uniform electric field, as shown.

• Does the electric field do positive or negative work on the proton?
• Does the electric potential energy of the proton increase or decrease?
• Does our force do positive or negative work?
• Does the proton move to a higher or lower potential?
Example

Consider a positive and a negative charge, freely moving in a uniform electric field. True or false?

(a) Positive charge moves to points with lower potential.
(b) Negative charge moves to points with lower potential.
(c) Positive charge moves to a lower potential energy position.
(d) Negative charge moves to a lower potential energy position.

(a) True  
(b) False  
(c) True  
(d) True
Conservative forces

The potential difference between two points is independent of the path taken to calculate it: electric forces are “conservative”.

\[ \Delta V = V_f - V_i = -\frac{W}{q_0} = \frac{\Delta U}{q_0} = -\int_{i}^{f} \vec{E} \cdot d\vec{s} \]
Electric Potential of a Point Charge

\[ V = -\int_{i}^{f} \vec{E} \cdot d\vec{s} = -\int_{\infty}^{P} E \, ds = \]

\[ = -\int_{\infty}^{R} \frac{kQ}{r^2} \, dr = + \frac{kQ}{r} \bigg|_{\infty}^{R} = + \frac{kQ}{R} \]

Note: if \( Q \) were a negative charge, \( V \) would be negative
Electric Potential of Many Point Charges

- Electric potential is a SCALAR not a vector.
- Just calculate the potential due to each individual point charge, and add together! (Make sure you get the SIGNS correct!)

\[ V = \sum_{i} k \frac{q_i}{r_i} \]
Electric potential and electric potential energy

\[ \Delta U = W_{app} = q\Delta V \]

What is the potential energy of a dipole?

- First bring charge \( +Q \): no work involved, no potential energy.
- The charge \( +Q \) has created an electric potential everywhere, \( V(r) = kQ/r \)
- The work needed to bring the charge \( -Q \) to a distance \( a \) from the charge \( +Q \) is \( W_{app} = U = (-Q)V = (-Q)(+kQ/a) = -kQ^2/a \)
- The dipole has a negative potential energy equal to \( -kQ^2/a \): we had to do negative work to build the dipole (and the electric field did positive work).
Potential Energy of A System of Charges

- 4 point charges (each +Q and equal mass) are connected by strings, forming a square of side L
- If all four strings suddenly snap, what is the kinetic energy of each charge when they are very far apart?
- Use conservation of energy:
  - Final kinetic energy of all four charges = initial potential energy stored = energy required to assemble the system of charges

Do this from scratch!
Potential Energy of A System of Charges: Solution

• No energy needed to bring in first charge: $U_1 = 0$
• Energy needed to bring in 2nd charge: $U_2 = QV_1 = \frac{kQ^2}{L}$
• Energy needed to bring in 3rd charge =
  
  \[ U_3 = QV = Q(V_1 + V_2) = \frac{kQ^2}{L} + \frac{kQ^2}{\sqrt{2}L} \]
• Energy needed to bring in 4th charge =
  
  \[ U_4 = QV = Q(V_1 + V_2 + V_3) = \frac{2kQ^2}{L} + \frac{kQ^2}{\sqrt{2}L} \]

Total potential energy is sum of all the individual terms shown on left hand side = $\frac{kQ^2}{L} (4 + \sqrt{2})$

So, final kinetic energy of each charge = $\frac{kQ^2}{4L} (4 + \sqrt{2})$
Summary:

- **Electric potential**: work needed to bring +1C from infinity; units \( V = \text{Volt} \)
- Electric potential uniquely defined for every point in space - independent of path!
- Electric potential is a **scalar** — add contributions from individual point charges
- We calculated the electric potential produced by a single charge: \( V = kq/r \), and by continuous charge distributions: \( V = \int k dq/r \)
- **Electric potential energy**: work used to build the system, charge by charge. Use \( W = qV \) for each charge.