Lecture 25
Optics: Images
Thin Lenses

For small angles and thin lenses,

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f}$$

Convergent: $f$ positive

Divergent: $f$ negative

Lens maker’s equation

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$
Images due to lenses:

- An object placed **beyond a convergent lenses’ focal point**, will produce a **real, inverted image** on the other side of the lens. This is the principle used in projectors.

- An object placed **between a convergent lens and its focal point** will produce a **virtual image** on the same side as the object.

- Divergent lenses **always produce a virtual image** on the same side as the object.

- Real images have \( i \) positive in formulas, virtual images have \( i \) negative.
Locating images by drawing rays:

- A ray of direction initially parallel to the axis will pass through the focal point.

- A ray that initially has a direction that passes through the focal point will emerge parallel to the central axis.

- A ray going through the center of the lens will be undeflected.

- The image of a point appears where all rays emanating from a point intersect.
Example

• An object 1.2cm high is placed 4cm from a bi-convex lens with \( r_1 = 10 \text{cm} \) and \( r_2 = 15 \text{cm} \). Find the position and size of the image.

• A second lens of focal length +6cm is placed 12cm to the right of the first lens. Find the position and size of the new image.
Images from spherical mirrors

Consider an object placed between the focal point and the mirror. It will produce a virtual image behind the mirror.

When the object is at the focal point the image is produced at infinity.

If the object is beyond the focal point, a real image forms at a distance $i$ from the mirror.

\[
\frac{1}{p} + \frac{1}{i} = \frac{1}{f}
\]

\[
m = -\frac{i}{p}
\]

Check the signs!!
For small angles and thin lenses,

\[
\frac{1}{p} + \frac{1}{i} = \frac{1}{f}
\]

Convergent: \( f \) positive

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Lens maker’s equation

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\frac{1}{f} = (n - 1) \left( \frac{1}{r_1} - \frac{1}{r_2} \right)
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Images due to lenses:

- An object placed **beyond a convergent lenses’ focal point**, will produce a **real, inverted image** on the other side of the lens. This is the principle used in slide projectors.

- An object placed **between a convergent lens and its focal point** will produce a **virtual image** on the same side as the object.

- Divergent lenses **always produce a virtual image on the same side** as the object.

- Real images have $i$ positive in formulas, virtual images have $i$ negative.
Example

• An object 2cm high is placed 4cm from a bi-convex lens with \( r_1 = 10cm \) and \( r_2 = 15cm \), and index of refraction \( n=1.5 \). Find the position and size of the image.

• A second lens of focal length +6cm is placed 12cm to the right of the first lens. Find the position and size of the new image.
Optical Instruments: the human eye

The human eye consists of a variable-geometry lens (crystalline) which produces a real image on a “screen” (retina) which is transmitted to the brain via the optical nerve.

The crystalline automatically adjusts itself so we see well any object placed between infinity and a distance called “near point” (about 25cm for a typical 20 year old). The “image distance” is the eye diameter~2cm.
Optical instruments: combination of several (thin) lenses

If lenses are very close, the compound lens has $1/f \sim 1/f_1 + 1/f_2$
Corrective Glasses

A farsighted person needs a convergent lens.

A nearsighted person needs a divergent lens.

The “power” of a lens is measured in **dioptres**: \( P = 1/f \) with \( f \) is in m. Glasses with -6D are divergent glasses with \( f = -1/6D = -0.17m = -17cm \). The dioptres add! Two lenses have \( 1/f = 1/f_1 + 1/f_2 \rightarrow D = D_1 + D_2 \).
Magnifying lens

The magnification of an object is \( m = \frac{i}{p} \approx \frac{i\theta}{h} \), but \( i = \) eye diameter.

Maximum magnification: \( m \approx \frac{2\text{cm}}{25\text{cm}} \) (!?)

We’d like to make \( p \) smaller (move the object closer). We use a magnifying lens to produce a (larger) image than our eye can see:

Angular magnification (different from lateral): \( m_\theta = \frac{\theta'}{\theta} \).

\[
\theta = \frac{h}{25\text{cm}} \quad \theta' \approx \frac{h}{f} \quad m_\theta = \frac{25\text{cm}}{f}
\]
Microscope:

To increase the magnification of a lens, one wants to have a short focal length. That means small radii of curvature (very curved lens). This, in turn implies a lot of aberration (one is immediately out of the thin lens approximation). A solution to this is obtained by combining two lenses. The resulting device is called microscope.

Object O is magnified by the objective:

\[ m = - \frac{i}{p} \]

And its image is magnified by the eyepiece:

\[ m_\theta = \frac{25cm}{f} \]

Total magnification:

\[ M = mm_\theta = - \frac{s}{f_{ob}} \frac{25cm}{f_{ey}} \]
Telescope:

Telescopes are arrangement of lenses that improve vision of objects very far away. They are configured like a microscope. However, the objective forms an image essentially at its focus, and therefore the eyepiece’s focus has to be placed at that same point.

The magnification is given by the ratio $\theta_{ey}/\theta_{ob}$, and since

$$\theta_{ob} = h'/f_{ob} \quad \theta_{ey} = h'/f_{ey}$$

$$m = -\frac{f_{ob}}{f_{ey}}$$

Refracting telescopes are of limited use (chromatic aberration). Reflecting telescopes built with mirrors are preferred in astronomy.
Example

The world’s largest refracting telescope is at the Yerkes Observatory of the University of Chicago at Williams Bay, Wisconsin. The objective has a diameter of 102cm and a focal length of 19.5m. The focal length of the eyepiece is 10cm. What is its magnifying power?

\[ m = -\frac{f_{ob}}{f_{ey}} = -\frac{19.5m}{0.1m} = -195 \]

Why so large (102cm)? Because the larger the objective, the more light it gathers.
Reflective telescopes

Keck observatory (Mauna Kea, Hawaii) and the Hale-Bopp comet.

Largest optical telescope, composed of 36 (!) hexagonal mirror segments performing as a single mirror 10m wide.