Physics 2102 Spring 2007
Lecture 17
Ch30:
Induction and Inductance I
Faraday’s Law

- A time varying magnetic flux creates an induced EMF.
- Definition of magnetic flux is similar to definition of electric flux.

\[ \Phi_B = \int_S \vec{B} \cdot \hat{n} \, dA \]

\[ EMF = -\frac{d\Phi_B}{dt} \]

- Take note of the MINUS sign!!
- The induced EMF acts in such a way that it OPPOSES the change in magnetic flux ("Lenz’s Law").
Example

- When the N pole approaches the loop, the flux “into” the loop (“downwards”) increases.
- The loop can “oppose” this change if a current were to flow clockwise, hence creating a magnetic flux “upwards.”
- So, the induced EMF is in a direction that makes a current flow clockwise.
- If the N pole moves AWAY, the flux “downwards” DECREASES, so the loop has a counter clockwise current!
Example

- A closed loop of wire encloses an area of 1 m\(^2\) in which in a uniform magnetic field exists at 30\(^0\) to the PLANE of the loop. The magnetic field is DECREASING at a rate of 1T/s. The resistance of the wire is 10 \(\Omega\).

- What is the induced current?

\[
\Phi_B = \int_S \vec{B} \cdot \vec{n} \, dA = \frac{BA}{2} 
\]

\[
\text{Is it } \quad \text{...clockwise or} \\
\text{...counterclockwise?}
\]

\[
i = \frac{EMF}{R} = \frac{A}{2R} \frac{dB}{dt} = \frac{(1m^2)}{2(10\Omega)} \frac{1T}{s} = 0.05A
\]
Example

• 3 loops are shown.
• $B = 0$ everywhere except in the circular region where $B$ is uniform, pointing out of the page and is increasing at a steady rate.

• Rank the 3 loops in order of increasing induced EMF.
  – (a) III, II, I
  – (b) III, (I & II are same)
  – (c) ALL SAME.

• Just look at the rate of change of ENCLOSED flux
• III encloses no flux and it does not change.
• I and II enclose same flux and it changes at same rate.
Example

- An infinitely long wire carries a constant current $i$ as shown.
- A square loop of side $L$ is moving towards the wire with a constant velocity $v$.
- What is the EMF induced in the loop when it is a distance $R$ from the loop?

Choose a “strip” of width $dx$ located as shown. Flux thru this “strip”

$$\Phi_B = \int_0^L \frac{\mu_0iL(dx)}{2\pi(R + x)}$$

$$= \left[ \frac{\mu_0iL}{2\pi} \ln(R + x) \right]_0^L$$

$$= \frac{\mu_0iL}{2\pi} \ln\left[ \frac{R + L}{R} \right]$$

$$EMF = -\frac{d\Phi_B}{dt}$$

$$= -\frac{\mu_0Li}{2\pi} \frac{d}{dt} \left\{ \ln\left[ 1 + \frac{L}{R} \right] \right\}$$
\[ EMF = -\frac{d\Phi_B}{dt} \]

\[ = -\frac{\mu_0 Li}{2\pi} \frac{d}{dt} \left\{ \ln \left[ 1 + \frac{L}{R} \right] \right\} \]

\[ = \frac{\mu_0 Li}{2\pi} \frac{dR}{dt} \left[ \frac{R}{R + L} \right] \frac{L}{R^2} \]

\[ = \frac{\mu_0 i}{2\pi} \nu \left[ \frac{L^2}{(R + L)R} \right] \]

What is the DIRECTION of the induced current?

- Magnetic field due to wire points INTO page and gets stronger as you get closer to wire
- So, flux into page is INCREASING
- Hence, current induced must be counter clockwise to oppose this increase in flux.
Example: the Generator

- A square loop of wire of side $L$ is rotated at a uniform frequency $f$ in the presence of a uniform magnetic field $B$ as shown.

- Describe the EMF induced in the loop.

$$
\Phi_B = \int_S \vec{B} \cdot \vec{n} dA
$$

$$
= BL^2 \cos(\theta)
$$

$$
EMF = - \frac{d\Phi_B}{dt} = BL^2 \frac{d\theta}{dt} \sin(\theta) = BL^2 (2\pi f) \sin(2\pi ft)
$$
Example: Eddy Currents

• A non-magnetic (e.g. copper, aluminum) ring is placed near a solenoid.

• What happens if:
  – There is a steady current in the solenoid?
  – The current in the solenoid is suddenly changed?
  – The ring has a “cut” in it?
  – The ring is extremely cold?
Another Experimental Observation

• Drop a non-magnetic pendulum (copper or aluminum) through an inhomogeneous magnetic field
• What do you observe? Why? (Think about energy conservation!)

Pendulum had kinetic energy
What happened to it?
Isn’t energy conserved??
Some interesting applications

MagLev train relies on Faraday’s Law: currents induced in non-magnetic rail tracks repel the moving magnets creating the induction; result: levitation!

Guitar pickups also use Faraday’s Law - a vibrating string modulates the flux through a coil hence creating an electrical signal at the same frequency.
Example: the ignition coil

- The gap between the spark plug in a combustion engine needs an electric field of $\sim 10^7$ V/m in order to ignite the air-fuel mixture. For a typical spark plug gap, one needs to generate a potential difference $> 10^4$ V!
- But, the typical EMF of a car battery is 12 V. So, how does a spark plug work??

The “ignition coil” is a double layer solenoid:
- Primary: small number of turns -- 12 V
- Secondary: MANY turns -- spark plug

http://www.familycar.com/Classroom/ignition.htm
Another formulation of Faraday’s Law

- We saw that a time varying magnetic flux creates an induced EMF in a wire, exhibited as a current.
- Recall that a current flows in a conductor because of electric field.
- Hence, a time varying magnetic flux must induce an ELECTRIC FIELD!
- Closed electric field lines!!?? No potential!!

\[ \oint_C \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt} \]

Another of Maxwell’s equations!
To decide SIGN of flux, use right hand rule: curl fingers around loop, +flux -- thumb.
Example

- The figure shows two circular regions $R_1$ & $R_2$ with radii $r_1 = 1\, \text{m}$ & $r_2 = 2\, \text{m}$. In $R_1$, the magnetic field $B_1$ points out of the page. In $R_2$, the magnetic field $B_2$ points into the page.
- Both fields are uniform and are DECREASING at the SAME steady rate $= 1\, \text{T/s}$.
- Calculate the “Faraday” integral $\oint C \vec{E} \cdot d\vec{s}$ for the two paths shown.

Path I: $\oint_{C_1} \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt} = -(\pi r_1^2)(-1\, \text{T}/\text{s}) = +3.14\, \text{V}$

Path II: $\oint_{C_2} \vec{E} \cdot d\vec{s} = -\left[ (\pi r_1^2)(-1\, \text{T}/\text{s}) + (\pi r_2^2)(-1\, \text{T}/\text{s}) \right] = +9.42\, \text{V}$
Example

• A long solenoid has a circular cross-section of radius $R$.
• The current through the solenoid is increasing at a steady rate $di/dt$.
• Compute the variation of the electric field as a function of the distance $r$ from the axis of the solenoid.

First, let’s look at $r < R$:

$$E(2\pi r) = (\pi r^2) \frac{dB}{dt}$$

$$E = \frac{r dB}{2 dt}$$

Next, let’s look at $r > R$:

$$E(2\pi r) = (\pi R^2) \frac{dB}{dt}$$

$$E = \frac{R^2 dB}{2r dt}$$
Example (continued)

\[ E = \frac{r \ dB}{2 \ dt} \]

\[ E = \frac{R^2 \ dB}{2r \ dt} \]
Summary

Two versions of Faraday’s law:

- A varying magnetic flux produces an EMF:

\[ EMF = - \frac{d\Phi_B}{dt} \]

- A varying magnetic flux produces an electric field:

\[ \oint_C \vec{E} \cdot d\vec{s} = - \frac{d\Phi_B}{dt} \]