Physics 2102
Lecture 15
Biot-Savart Law

Jean-Baptiste Biot (1774-1862)
Felix Savart (1791–1841)
What Are We Going to Learn?
A Road Map

• Electric charge
  ➞ Electric force on other electric charges
  ➞ Electric field, and electric potential
• Moving electric charges: current
• Electronic circuit components: batteries, resistors, capacitors
• Electric currents ➞ Magnetic field
  ➞ Magnetic force on moving charges
• Time-varying magnetic field ➞ Electric Field
• More circuit components: inductors.
• Electromagnetic waves ➞ light waves
• Geometrical Optics (light rays).
• Physical optics (light waves)
Electric Current: A Source of Magnetic Field

- Observation: an electric current creates a magnetic field
- Simple experiment: hold a current-carrying wire near a compass needle!
Yet Another Right Hand Rule!

- Point your thumb along the direction of the current in a straight wire.
- The magnetic field created by the current consists of circular loops directed along your curled fingers.
- The magnetic field gets weaker with distance.
- You can apply this to ANY straight wire (even a small differential element!)
- What if you have a curved wire? Break into small elements.
Superposition

• Magnetic fields (like electric fields) can be “superimposed” -- just do a vector sum of $B$ from different sources

• The figure shows four wires located at the 4 corners of a square. They carry equal currents in directions indicated

• What is the direction of $B$ at the center of the square?
Biot-Savart Law

When we computed the electric field due to charges we used Coulomb’s law. If one had a large irregular object, one broke it into infinitesimal pieces and computed,

\[ d\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{dq}{r^2} \hat{r} \]

Which we write as,

\[ d\vec{E} = \frac{dq}{4\pi\varepsilon_0} \frac{\vec{r}}{r^3} \]

If you wish to compute the magnetic field due to a current in a wire, you use the law of Biot and Savart.
The Biot-Savart Law

- Quantitative rule for computing the magnetic field from any electric current
- Choose a differential element of wire of length $dL$ and carrying a current $i$
- The field $dB$ from this element at a point located by the vector $r$ is given by the Biot-Savart Law

$$dB = \frac{\mu_0 i dL \times \vec{r}}{4\pi r^3}$$

Compare with

$$dE = \frac{dq \vec{r}}{4\pi \varepsilon_0 r^3}$$

$\mu_0 = 4\pi \times 10^{-7}$ Tm/A (permeability constant)
Biot-Savart Law

- An infinitely long straight wire carries a current $i$.
- Determine the magnetic field generated at a point located at a perpendicular distance $R$ from the wire.
- Choose an element $ds$ as shown.
- Biot-Savart Law: $dB$ points INTO the page.
- Integrate over all such elements.

\[
dB = \frac{\mu_0 i ds \times \vec{r}}{4\pi r^3}
\]

\[
B = \frac{\mu_0 i}{4\pi} \int_{-\infty}^{\infty} ds (r \sin \theta) \frac{1}{r^3}
\]
Field of a Straight Wire

\[
\begin{align*}
\mathbf{d}\mathbf{B} &= \frac{\mu_0 \mathbf{i} ds \times \mathbf{r}}{4\pi r^3} \\
\mathbf{d}\mathbf{B} &= \frac{\mu_0 \mathbf{i} ds (r \sin \theta)}{4\pi r^3}
\end{align*}
\]

\[
\sin \theta = \frac{R}{r} \quad r = (s^2 + R^2)^{1/2}
\]

\[
B = \frac{\mu_0 \mathbf{i}}{4\pi} \int_{-\infty}^{\infty} ds \frac{(r \sin \theta)}{r^3}
= \frac{\mu_0 \mathbf{i}}{4\pi} \int_{-\infty}^{\infty} \frac{Rds}{(s^2 + R^2)^{3/2}}
= \frac{\mu_0 \mathbf{i}}{2\pi} \int_{0}^{\infty} \frac{Rds}{(s^2 + R^2)^{3/2}}
= \frac{\mu_0 \mathbf{i} R}{2\pi} \left[\frac{s}{R^2 \left(s^2 + R^2\right)^{1/2}}\right]_{0}^{\infty}
= \frac{\mu_0 \mathbf{i}}{2\pi R}
\]
Example : A Practical Matter

A power line carries a current of 500 A. What is the magnetic field in a house located 100 m away from the power line?

\[ B = \frac{\mu_0 i}{2\pi R} \]

\[ = \left( \frac{4\pi \times 10^{-7} T \cdot m / A}{2\pi} \right)(500 A) \]

\[ = \frac{1 \mu T!!}{2\pi (100 m)} \]

Recall that the earth’s magnetic field is \( \sim 10^{-4} T = 100 \mu T \)
**Biot-Savart Law**

- A circular arc of wire of radius $R$ carries a current $i$.
- What is the magnetic field at the center of the loop?

\[
d\vec{B} = \frac{\mu_0}{4\pi} \frac{id\vec{s} \times \vec{r}}{r^3}
\]

\[
d\vec{B} = \frac{\mu_0}{4\pi} \frac{id\vec{s}R}{R^3} = \frac{\mu_0}{4\pi} \frac{iRd\phi}{R^2}
\]

\[
B = \frac{\mu_0}{4\pi} \int \frac{id\phi}{R} = \frac{\mu_0 i\Phi}{4\pi R}
\]

**Direction of B?? Not another right hand rule?!**

**TWO right hand rules!:**

- If your thumb points along the CURRENT, your fingers will point in the same direction as the FIELD.
- If you curl our fingers around the direction of CURRENT, your thumb points along FIELD!
Forces between wires

Magnetic field due to wire 1 where the wire 2 is,

\[ B_1 = \frac{\mu_0 I_1}{2\pi a} \]

Force on wire 2 due to this field,

\[ F_{21} = L I_2 B_1 = \frac{\mu_0 LI_1 I_2}{2\pi a} \]
Summary

- **Magnetic fields exert forces on moving charges:**
  - The force is perpendicular to the field and the velocity.
  - A current loop is a magnetic dipole moment.
  - Uniform magnetic fields exert torques on dipole moments.

- **Electric currents produce magnetic fields:**
  - To compute magnetic fields produced by currents, use Biot-Savart’s law for each element of current, and then integrate.
  - Straight currents produce circular magnetic field lines, with amplitude \( B = \mu_0 i / 2\pi r \) (use right hand rule for direction).
  - Circular currents produce a magnetic field at the center (given by another right hand rule) equal to \( B = \mu_0 i \Phi / 4\pi r \).

- **Wires currying currents produce forces on each other:** parallel currents attract, antiparallel currents repel.