Physics 2102
Lecture 14
Ch28: Magnetic Forces on Current Wires
Crossed Fields \(E\) vs. \(B\)

\[
F_E = qE \\
F_E = ma \implies y = qEL^2/(2mv^2) \\
F_B = vqBF_E \\
F_B = F_E \implies y = 0 \implies v = E/B
\]
Magnetic force on a wire.

\[ q = i \, t = i \frac{L}{v_d} \]

\[ \vec{F} = q \vec{v}_d \times \vec{B} \]

\[ \vec{F} = q \frac{i \vec{L}}{q} \times \vec{B} = i \vec{L} \times \vec{B} \]

Note: If wire is not straight, compute force on differential elements and integrate:

\[ d\vec{F} = i \, d\vec{L} \times \vec{B} \]
Example

Wire with current $i$.

Magnetic field out of page.

What is net force on wire?

\[ F_1 = F_3 = iLB \]

\[ dF = iBdL = iBRd\theta \]

By symmetry, $F_2$ will only have a vertical component,

\[ F_2 = \int_0^\pi \sin(\theta)dF = iBR \int_0^\pi \sin(\theta)d\theta = 2iBR \]

\[ F_{\text{total}} = F_1 + F_2 + F_3 = iLB + 2iRB + iLB = 2iB(L + R) \]

Notice that the force is the same as that for a straight wire, and this would be true no matter what the shape of the central segment!
Example 4: The Rail Gun

- Conducting projectile of length 2cm, mass 10g carries constant current 100A between two rails.
- Magnetic field $B = 100T$ points outward.
- Assuming the projectile starts from rest at $t = 0$, what is its speed after a time $t = 1s$?

- Force on projectile: $F = iL B$  
  (from $F = iL \times B$)
- Acceleration: $a = iLB/m$  
  (from $F = ma$)
- $\nu(t) = iLBt/m$  
  (from $\nu = \nu_0 + at$)

$$\begin{align*}
  &= (100A)(0.02m)(100T)(1s)/(0.01kg) = 2000m/s \\
  &= 4,473mph = MACH 8!
\end{align*}$$
Torque on a Current Loop:

Rectangular coil: \( A=ab \), current = \( i \)

Principle behind electric motors.

Net force on current loop = 0

But: Net torque is NOT zero!

\[ F_1 = F_3 = iaB \]

\[ F_\perp = F_1 \sin(\theta) \]

\[
\text{Torque} = |\tau| = F_\perp b = iabB \sin(\theta)
\]

For a coil with \( N \) turns,

\( \tau = NIAB \sin \theta \),

where \( A \) is the area of coil
Magnetic Dipole Moment

We just showed: \( \tau = NiAB \sin \theta \)

\( N = \) number of turns in coil
\( A = \) area of coil.

Define: magnetic dipole moment \( \mu \)

\[
\vec{\mu} = (NiA)\hat{n}
\]

\[
\vec{\tau} = \vec{\mu} \times \vec{B}
\]

As in the case of electric dipoles, magnetic dipoles tend to align with the magnetic field.

Right hand rule:
curl fingers in direction of current;
thumb points along \( \mu \)
Electric vs. Magnetic Dipoles

Electric dipole:
\[ \vec{D} = \vec{p} \times \vec{E} \]

Magnetic dipole:
\[ \vec{\tau} = \vec{\mu} \times \vec{B} \]

\[ \vec{\mu} = (NiA)\hat{n} \]