Physics 2102
Lecture 15
Magnetic fields

Aurora Borealis

“T’ll be back….

Star Quake on a Magnetar!
Use of Magnetic Fields in Your Everyday Life!
Electric fields are created:

- *microscopically*, by electric charges (fields) of elementary particles (electrons, protons)
- *macroscopically*, by adding the field of many elementary charges of the same sign

Magnetic fields are created:

- *microscopically*, by magnetic “moments” of elementary particles (electrons, protons, neutrons)
- *macroscopically*, by
  - adding many microscopic magnetic moments (magnetic materials); or by
  - electric charges that move (electric currents)
We know that an electric fields exists because it accelerates electric charges, with a force independent of the velocity of the charge, proportional to the electric charge: $F_E = qE$

We know that a magnetic field exists because it accelerates electric charges in a direction perpendicular to the velocity of the charge, with a magnitude proportional to the velocity of the charge and to the magnitude of the charge: $F_B = qv \times B$

Magnetic forces are perpendicular to both the velocity of charges and to the magnetic field (electric forces are parallel to the field).

Since magnetic forces are perpendicular to the velocity, they do no work! ($W = F \cdot r$)

Speed of particles moving in a magnetic field remains constant in magnitude, the direction changes. Kinetic energy is constant! (no work).
Circular Motion: Since magnetic force is transverse to motion, the natural movement of charges is circular.

$$F = ma = m\frac{v^2}{r}$$ for circular motion

Therefore \( q \nu B = \frac{mv^2}{r} \)

\[ r = \frac{mv}{qB} \]

In general, path is a helix (component of \( \nu \) parallel to field is unchanged).
Two charged ions A and B traveling with a constant velocity $v$ enter a box in which there is a uniform magnetic field directed out of the page. The subsequent paths are as shown. What can you conclude?

(a) Both ions are negatively charged.
(b) Ion $A$ has a larger mass than $B$.
(c) Ion $A$ has a larger charge than $B$.
(d) None of the above.

Same speed and $B$ for both masses.
So: ion with larger mass:charge ratio ($m/q$) moves in circle of larger radius. But that’s all we know!
Cathode Ray Tube  (Old TVs & Computer Monitors)

- Hot cathode emits electrons
- Get accelerated by positive plate
- Might be deflected using plates
- Produce point of light on screen.

In a magnetic field:

\[ \mathbf{B} \times \mathbf{v} \]

Dot shifts sideways.
Examples of Motion in Magnetic Fields

Aurora borealis (northern lights)

Synchrotron

Suppose you wish to accelerate charged particles as fast as you can.

Linear accelerator (long).

Fermilab, Batavia, IL (1km)
Magnetic force on a wire.

\[ q = i t = i \frac{L}{v_d} \]

\[ \vec{F} = q \vec{v}_d \times \vec{B} \]

\[ \vec{F} = q \frac{i \vec{L}}{q} \times \vec{B} = i \vec{L} \times \vec{B} \]

\[ \vec{F} = i \vec{L} \times \vec{B} \]

Note: If wire is not straight, compute force on differential elements and integrate:

\[ d\vec{F} = i \, d\vec{L} \times \vec{B} \]
Example

Wire with current $i$.
Magnetic field out of page.
What is net force on wire?

$$F_1 = F_3 = iLB$$
$$dF = iBdL = iBRd\theta$$

By symmetry, $F_2$ will only have a vertical component,

$$F_2 = \int_0^\pi \sin(\theta) dF = iBR \int_0^\pi \sin(\theta) d\theta = 2iBR$$

$$F_{\text{total}} = F_1 + F_2 + F_3 = iLB + 2iRB + iLB = 2iB(L + R)$$

Notice that the force is the same as that for a straight wire,
and this would be true no matter what the shape of the central segment!
Example 4: The Rail Gun

• Conducting projectile of length 2cm, mass 10g carries constant current 100A between two rails.

• Magnetic field $B = 100T$ points outward.

• Assuming the projectile starts from rest at $t = 0$, what is its speed after a time $t = 1s$?

• Force on projectile: $F = ILB$  

• Acceleration: $a = iLB/m$  

• $v(t) = iLBt/m$  

$= (100A)(0.02m)(100T)(1s)/(0.01kg) = 2000m/s$  
$= 4,473mph = MACH 8!$
Rail guns in the “Eraser” movie

"Rail guns are hyper-velocity weapons that shoot aluminum or clay rounds at just below the speed of light. In our film, we've taken existing stealth technology one step further and given them an X-ray scope sighting system," notes director Russell. "These guns represent a whole new technology in weaponry that is still in its infancy, though a large-scale version exists in limited numbers on battleships and tanks. They have incredible range. They can pierce three-foot thick cement walls and then knock a canary off a tin can with absolute accuracy. In our film, one contractor has finally developed an assault-sized rail gun. We researched this quite a bit, and the technology is really just around the corner, which is one of the exciting parts of the story."

Warner Bros., production notes, 1996.
http://movies.warnerbros.com/eraser/cmp/prodnotes.html#tech

Also: INSULTINGLY STUPID MOVIE PHYSICS: http://www.intuitior.com/moviephysics/
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Distribution is unlimited.
Torque on a Current Loop:

Rectangular coil: $A=ab$, current = $i$

Principle behind electric motors.

Net force on current loop = 0

But: Net torque is NOT zero!

$$F_1 = F_3 = iaB$$

$$F_\perp = F_1 \sin(\theta)$$

Torque = $|\tau| = F_\perp b = iabB \sin(\theta)$

For a coil with $N$ turns,

$\tau = NIAB \sin\theta,$

where $A$ is the area of coil
Magnetic Dipole Moment

We just showed: \( \tau = NiAB \sin \theta \)

\( N = \) number of turns in coil
\( A = \) area of coil.

Define: magnetic dipole moment \( \mu \)

\[
\vec{\mu} = (NaI)\hat{n}
\]

\[
\vec{\tau} = \vec{\mu} \times \vec{B}
\]

Right hand rule:

curl fingers in direction of current; thumb points along \( \mu \)

As in the case of electric dipoles, magnetic dipoles tend to align with the magnetic field.
Electric vs. Magnetic Dipoles

\[ \vec{p} = Qa \]

\[ \vec{E} \]

\[ \vec{\tau} = \vec{p} \times \vec{E} \]

\[ \vec{\mu} = (NiA)\hat{n} \]

\[ \vec{\tau} = \vec{\mu} \times \vec{B} \]