Lecture 31: MON 30 MAR
Ch.32.1–3: Maxwell’s equations

James Clerk Maxwell (1831-1879)
Maxwell I: Gauss’ Law for E-Fields:
charges produce electric fields, field lines start and end in charges

\[ \int \mathbf{E} \cdot d\mathbf{A} = \frac{q}{\varepsilon_0} \]

Electric field lines
Maxwell II: Gauss' law for B-Fields:

field lines are closed
or, there are no magnetic monopoles

\[ \oint_B \mathbf{B} \cdot d\mathbf{A} = 0 \]
Maxwell III: Ampere’s law:

Electric currents produce magnetic fields

\[ \oint_{C} B \cdot ds = \mu_0 i \]
Maxwell IV: Faraday’s law: changing magnetic fields produce (“induce”) electric fields

\[ \oint_C E \cdot ds = -\frac{d}{dt} \int_S B \cdot dA \]
Maxwell Equations I – IV:

\[ \oint_S E \cdot dA = q / \varepsilon_0 \]

\[ \oint_S B \cdot dA = 0 \]

\[ \oint_C B \cdot ds = \mu_0 i \]

\[ \oint_C E \cdot ds = -\frac{d}{dt} \oint_S B \cdot dA \]
In Empty Space with No Charge or Current

\[ \oint E \cdot dA = 0 \]

\[ \oint B \cdot dA = 0 \]

\[ \oint B \cdot ds = 0 \]

\[ \oint E \cdot ds = -\frac{d}{dt} \oint_s B \cdot dA \]

\( q=0 \)

\( i=0 \)

...very suspicious...!
Maxwell’s Displacement Current

If we are charging a capacitor, there is a current left and right of the capacitor.

Thus, there is the same magnetic field right and left of the capacitor, with circular lines around the wires.

But no magnetic field inside the capacitor?

With a compass, we can verify there is indeed a magnetic field, equal to the field elsewhere.

But there is no current producing it!?
**Maxwell’s Fix**

We calculate the magnetic field produced by the currents at left and at right using Ampere’s law:

\[ \oint_B \mathbf{B} \cdot d\mathbf{s} = \mu_0 i \]

We can write the current as:

\[
i = \frac{dq}{dt} = \frac{d(CV)}{dt} = C \frac{dV}{dt} = \varepsilon_0 A \frac{d(Ed)}{dt} = \varepsilon_0 \frac{d(EA)}{dt} = \varepsilon_0 \frac{d\Phi_E}{dt}
\]

\[
q = CV \quad C = \varepsilon_0 A/d \quad V = Ed \quad \Phi_E = \int \mathbf{E} \cdot d\mathbf{A} = EA
\]
Displacement Current

\[ \oint_C B \cdot ds \neq 0 \quad \text{Maxwell proposed it based on symmetry and math — no experiment!} \]

\[ \oint_C B \cdot ds = \mu_0 \varepsilon_0 \frac{d}{dt} \int_S E \cdot dA \]
Maxwell’s Equations I – V:

I. \[ \int_S E \cdot dA = \frac{q}{\varepsilon_0} \]

II. \[ \int_S B \cdot dA = 0 \]

III. \[ \oint_C B \cdot ds = \mu_0 \varepsilon_0 \frac{d}{dt} \int_S E \cdot dA + \mu_0 i \]

IV. \[ \oint_C E \cdot ds = -\frac{d}{dt} \int_S B \cdot dA \]
Maxwell Equations in Empty Space:

\[ \oint_S E \cdot dA = 0 \]
\[ \oint_S B \cdot dA = 0 \]
\[ \oint_C B \cdot ds = \mu_0 \varepsilon_0 \frac{d}{dt} \oint_S E \cdot dA \]
\[ \oint_C E \cdot ds = -\frac{d}{dt} \oint_S B \cdot dA \]
Maxwell, Waves and Light

A solution to the Maxwell equations in empty space is a “traveling wave”...

\[ \int B \cdot ds = \mu_0 \varepsilon_0 \frac{d}{dt} \int E \cdot dA \]
\[ \int E \cdot ds = -\frac{d}{dt} \int B \cdot dA \]

Electric and magnetic “forces” can travel!

\[ \frac{d^2 E}{dx^2} = -\mu_0 \varepsilon_0 \frac{d^2 E}{dt^2} \Rightarrow E = E_0 \sin k(x - ct) \]

\[ c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8 \text{ m/s} \]

The “electric” waves travel at the speed of light!?

Light itself is a wave of electricity and magnetism!?
Electromagnetic waves

First person to prove that electromagnetic waves existed: Heinrich Hertz (1875-1894)

First person to use electromagnetic waves for communications: Guglielmo Marconi (1874-1937), 1909 Nobel Prize

(first transatlantic commercial wireless service, Nova Scotia, 1909)
Electromagnetic Waves: One Velocity, Many Wavelengths!

with frequencies measured in “Hertz” (cycles per second) and wavelength in meters.

http://imagers.gsfc.nasa.gov/ems/
http://www.astro.uiuc.edu/~kaler/sow/spectra.html
How do E&M Waves Travel?

Is there an “ether” they ride on? Michelson and Morley looked and looked, and decided it wasn’t there. How do waves travel???

Electricity and magnetism are “relative”: Whether charges move or not depends on which frame we use...

This was how Einstein began thinking about his “theory of special relativity”...

We’ll leave that theory for later.
Summary

• Changing electric fields produce (induce) magnetic fields: displacement currents.

• Maxwell’s laws allow us to calculate electric and magnetic fields everywhere in space if we are given the sources: electric charges and currents.

• If there are no sources, we can still have electric and magnetic fields as electromagnetic waves, which travel at the speed of light.