Physics 2102
Lecture: 11 FRI 06 FEB
Capacitance I

25.1–3
Tutoring Lab Now Open Tuesdays

- Lab Location: 102 Nicholson (across the hall from class)
- Lab Hours:
  MTWT: 12:00N–5:00PM
  F: 12:N–3:00PM
Capacitors and Capacitance

Capacitor: any two conductors, one with charge $+Q$, other with charge $-Q$

Potential DIFFERENCE between conductors = $V$

$$Q = CV \text{ where } C = \text{capacitance}$$

Units of capacitance:
Farad (F) = Coulomb/Volt

Uses: storing and releasing electric charge/energy.
Most electronic capacitors:
- micro-Farads ($\mu$F),
- pico-Farads (pF) — $10^{-12}$ F
New technology:
compact 1 F capacitors
Capacitance

- Capacitance depends only on GEOMETRICAL factors and on the MATERIAL that separates the two conductors.
- e.g. Area of conductors, separation, whether the space in between is filled with air, plastic, etc.

(We first focus on capacitors where gap is filled by AIR!)
Parallel Plate Capacitor

We want \textit{capacitance}: \( C = \frac{Q}{V} \)

E field between the plates: (Gauss’ Law)

\[
E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}
\]

Relate \( E \) to potential difference \( V \):

\[
V = \int_0^d \vec{E} \cdot d\vec{x} = \int_0^d \frac{Q}{\varepsilon_0 A} \, dx = \frac{Qd}{\varepsilon_0 A}
\]

What is the capacitance \( C \)?

\[
C = \frac{Q}{V} = \frac{\varepsilon_0 A}{d}
\]

Units:

\[
\text{[C]} \left( \frac{\text{m}^2}{\text{Nm}^2} \right) = \text{[C]} \left( \frac{\text{m}^2}{\text{Nm}} \right) = \text{[C]} \left( \frac{\text{J}}{\text{C}} \right) = \text{[C]} \left( \frac{1}{\text{V}} \right)
\]
Capacitance and Your iPhone!

A capacitive sensor is a solid-state sensor made using standard pc-board or flex circuit technology. A finger on top of a grid of conductive traces changes the capacitance of the nearest traces. This change in trace capacitance can be measured, and finger position can be computed.

$$C = \frac{Q}{V} = \frac{\varepsilon_0 A}{d}$$
Parallel Plate Capacitor — Example

• A huge parallel plate capacitor consists of two square metal plates of side 50 cm, separated by an air gap of 1 mm.

• What is the capacitance?

$$C = \varepsilon_0 \frac{A}{d}$$

$$= (8.85 \times 10^{-12} \text{ F/m})(0.25 \text{ m}^2)/(0.001 \text{ m})$$

$$= 2.21 \times 10^{-9} \text{ F}$$

(Very Small!!)

Lesson: difficult to get large values of capacitance without special tricks!

Units:

$$\left[ \frac{C^2}{Nm^2} \frac{m^2}{m} \right] = \left[ \frac{C^2}{Nm} \right] = \left[ \frac{CC}{J} \right] = \left[ \frac{C}{V} \right] \equiv [F] = \text{Farad}$$
Isolated Parallel Plate Capacitor

- A parallel plate capacitor of capacitance $C$ is charged using a battery.
- Charge = $Q$, potential difference = $V$.
- Battery is then disconnected.
- If the plate separation is INCREASED, does Potential Difference $V$:

(a) Increase? ★
(b) Remain the same?
(c) Decrease?

- $Q$ is fixed!
- $C$ decreases ($=\varepsilon_0 A/d$)
- $V=Q/C$; $V$ increases.
A parallel plate capacitor of capacitance $C$ is charged using a battery.

Charge = $Q$, potential difference = $V$.

Plate separation is INCREASED while battery remains connected.

Does the Electric Field Inside:
(a) Increase?
(b) Remain the Same?
(c) Decrease?

- $V$ is fixed by battery!
- $C$ decreases ($=\varepsilon_0 A/d$)
- $Q=CV$; $Q$ decreases
- $E = Q/\varepsilon_0 A$ decreases
What is the electric field inside the capacitor? (Gauss’ Law)

\[ E = \frac{Q}{4\pi\varepsilon_0 r^2} \]

Relate \( E \) to potential difference between the plates:

\[ V = \int_{a}^{b} \vec{E} \cdot d\vec{r} = \int_{a}^{b} \frac{kQ}{r^2} dr = \left[ -\frac{kQ}{r} \right]_{a}^{b} = kQ \left[ \frac{1}{a} - \frac{1}{b} \right] \]
Spherical Capacitor

What is the capacitance?

\[ C = \frac{Q}{V} = \frac{Q}{4\pi \varepsilon_0 \left[ \frac{1}{a} - \frac{1}{b} \right]} \]

Radius of outer plate = \( b \)
Radius of inner plate = \( a \)

Concentric spherical shells:
Charge \( +Q \) on inner shell,
-\( Q \) on outer shell

Isolated sphere: let \( b \gg a \),

\[ C = 4\pi \varepsilon_0 a \]
Cylindrical Capacitor

What is the electric field in between the plates? Gauss’ Law!

\[ E = \frac{Q}{2\pi \varepsilon_0 rL} \]

Relate \( E \) to potential difference between the plates:

\[ V = \int_a^b \vec{E} \cdot d\vec{r} \]

\[ = \int_a^b \frac{Q}{2\pi \varepsilon_0 rL} dr = \left[ \frac{Q \ln r}{2\pi \varepsilon_0 L} \right]_a^b \]

\[ = \frac{Q}{2\pi \varepsilon_0 L} \ln\left(\frac{b}{a}\right) \]

Radius of outer plate = \( b \)
Radius of inner plate = \( a \)
Length of capacitor = \( L \)
+\( Q \) on inner rod, –\( Q \) on outer shell
Any two charged conductors form a capacitor.

Capacitance: \( C = \frac{Q}{V} \)

Simple Capacitors:

- Parallel plates: \( C = \varepsilon_0 \frac{A}{d} \)
- Spherical: \( C = 4\pi \varepsilon_0 \frac{ab}{b-a} \)
- Cylindrical: \( C = 2\pi \varepsilon_0 \frac{L}{\ln(b/a)} \)