Single Summation Ring Function Formulae
for the Rotationally Invariant Coordinate Systems
which $R$-Separate the Newton/Coulomb Interaction

Howard S. Cohl
Department of Physics and Astronomy, Louisiana State University

1. Introduction

A great many physics problems require the accurate determination of an either an electromagnetic or a gravitational field. The field generated by a non-trivial isolated mass/charge distribution, $\rho(x)$, for the most part, can be adequately described by classical electromagnetism or through Newtonian gravity. And therefore, can be derived from a scalar potential function, $\Phi(x)$. From a mathematical viewpoint, there are two methods for obtaining the potential: by solving the three-variable Poisson equation, $\nabla^2 \Phi(x) = 4\pi \rho(x)$, where $\nabla^2$ is the 3D Laplace operator, or by evaluating the integral expression for the potential, namely $\Phi(x) = -\int_V \rho(x') |x-x'|^{-1} d^3x'$, where $V$ represents the volume of integration and $|x-x'|^{-1}$ is the fundamental solution or the infinite extent Green's function for the Laplace equation, $\nabla^2 \Phi(x) = 0$. Double summation expansion formulae for $|x-x'|^{-1}$ exist in all coordinate systems which are known to be $R$-separable for the Laplace equation. Single summation expansion formulae are known to exist as well. In this talk, I describe a new quartet of single-summation ring function formulae; their relevance to the $(\nu, \mu) = (-\frac{1}{2}, 0)$ symmetric point for the associated Legendre functions $P_\nu^\mu(z)$ and $Q_\nu^\mu(z)$; implications for generalized axisymmetric potential theory; and a new twist in the interpretation of the multipole expansion.