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PQE, 03 January 2006, Snowbird

Not Shown: MA.Can, A.Chiruvelli, GA.Durkin, M.Erickson, L.Florescu, M.Florescu, KT.Kapale, SJ.Olsen, S.Thanvantri, Z.Wu
Two Roads to C-NOT

I. Enhance Nonlinear Interaction with a Cavity or EIT — Kimble, Walther, Lukin, et al.

II. Exploit Nonlinearity of Measurement — Knill, LaFlamme, Milburn, Franson, et al.
WHY IS A KERR NONLINEARITY LIKE A PROJECTIVE MEASUREMENT?
The success probability is less than 1 (namely 1/8).

The input state is constrained to be a superposition of 0, 1, and 2 photons only.

Conditioned on a detector coincidence in $D_1$ and $D_2$.

Effective $\kappa = 1/8$ → 22 Orders of Magnitude Improvement!

$|\psi_{in}\rangle = \sum_{n=0}^{2} c_n |n\rangle |1\rangle$

P. Kok, H. Lee, and JPD, PRA 66 (2003) 063814
A Revolution in Nonlinear Optics at the Few Photon Level: No Longer Limited by the Nonlinearities We Find in Nature!

Projective Measurement Yields Effective "Kerr"!

\[ Q = \frac{\pi \hbar}{2} (5 \hat{n} - \hat{n}^2) \]

KLM CSIGN Hamiltonian

\[ Q = \frac{\pi \hbar}{2} (3 + a_b^\dagger (1 - \hat{n}_b) + (1 - \hat{n}_b) a_b) \hat{n}_a \]

Franson CNOT Hamiltonian
We call the state of the form $|N, \phi > + |\phi, N >$ the NOON state, and the High NOON state a large $N$. 

\[ \frac{1 + \cos \varphi}{2} \] uncorrelated

\[ \frac{1 + \cos N\varphi}{2} \] correlated

$\varphi = kx$

$\Delta \varphi: \frac{1}{\sqrt{N}} \rightarrow \frac{1}{N}$
FROM QUANTUM INTERFEROMETER TO QUANTUM LITHOGRAPHY

N-Photon Absorbing Lithographic Resist

\[ \langle \psi | a_+^N a^N | \psi \rangle \]

\[ \begin{array}{c} \frac{1 + \cos \varphi}{2} \quad \text{uncorrelated} \\ \frac{1 + \cos N \varphi}{2} \quad \text{correlated} \end{array} \]

\[ \varphi = kx \]
\[ \varphi \rightarrow N \varphi \]
\[ \lambda \rightarrow \lambda/N \]
Showdown at High-NOON!

How do we make NOON!?

\[ |N,0\rangle + |0,N\rangle \]

With a large cross-Kerr nonlinearity!*

\[ \mathcal{H} = \kappa \, a^\dagger a \: b^\dagger b \]

This is not practical! —
need \( \kappa = \pi \) but \( \kappa = 10^{-22} \)!

Projective Measurements to the Rescue

Probability of success: \( \frac{3}{64} \)

Best we found: \( \frac{3}{16} \) Not Efficient!

De Broglie wavelength of a non-local four-photon state

Philip Walther\textsuperscript{1}, Jian-Wei Pan\textsuperscript{1,2}, Markus Aspelmeyer\textsuperscript{1}, Rupert Ursin\textsuperscript{1}, Sara Gasparoni\textsuperscript{1,2} & Anton Zeilinger\textsuperscript{1,2}

Super-resolving phase measurements with a multiphoton entangled state

M. W. Mitchell, J. S. Lundeen & A. M. Steinberg
What’s New with NOON States?

KT Kapale & JPD,
A Bootstrapping Approach for Generating Maximally Path-Entangled Photon States,
[quant-ph/0612196].

NM VanMeter, P Lougovski,
DB Uskov, K Kieling, J Eisert, JPD,
A General Linear-Optical Quantum State Generator,
[quant-ph/0612154].

Durkin GA, Dowling JP, Local and Global Distinguishability in Quantum Interferometry,
[quant-ph/0607088].
High-NOON Meets Phaseonium
With sufficiently high cross-Kerr nonlinearity, NOON generation possible.

Implementation via Phaseonium

\[ |\psi_{1(2)}\rangle_{ab} = \frac{1}{\sqrt{2}} \left[ |N\rangle_a |0\rangle_b \pm e^{i\xi N_0} |0\rangle_a |N\rangle_b \right] \]

Gerry and Campos, PRA 64 063814 (2001)
Two possible methods

• As a high-refractive index material to obtain the large phase shifts
  - Problem: Requires entangled phaseonium

• As a cross-Kerr nonlinearity
  - Problem: Does not offer required phase shifts of $\pi$ as yet (experimentally)
Phaseonium for High Index of Refraction

With larger density high index of refraction can be obtained

$N = 10^{15} \text{ cm}^{-3}$  \hspace{1cm} $\text{Re}(\chi) = 100 \text{ cm}^{-3}$

$n = 10 \text{ cm}^{-3}$
The needed large phase-shift of $\pi$ can be obtained via the phaseonium as a high refractive index material.

However, the control required by the Quantum Fredkin gate necessitates the atoms be in the GHZ state between level $a$ and $b$ Which could be possible for upto 1000 atoms.

Question: Would 1000 atoms give sufficiently high refractive index?
Cross-Kerr nonlinearities via Phaseonium have been shown to impart phase shifts of 7° controlled via single photon.

One really needs to input a smaller NOON as a control for the QFG as opposed to a single photon with $N=30$ roughly to obtain phase shift as large as $\pi$.

This suggests a bootstrapping approach.

In the presence of single signal photon, and the strong drive a weak probe field experiences a phase shift.
Implementation of QFG via Cavity QED

Ramsey Interferometry for atom initially in state $b$.

Dispersive coupling between the atom and cavity gives required conditional phase shift

$$|\psi_{1(2)}\rangle_{ab} = \frac{1}{\sqrt{2}} [ |N\rangle_a |0\rangle_b \pm e^{i\xi N_0} |0\rangle_a |N\rangle_b ]$$
Low-NOON via Entanglement
Swapping: The NOON gun

- Single photon gun of Rempe PRL 85 4872 (2000) and Fock state gun of Whaley group quant-ph/0211134 could be extended to obtain a NOON gun from atomic GHZ states.

- GHZ states of few 1000 atoms can be generated in a single step via (I) Agarwal et al. PRA 56 2249 (1997) and (II) Zheng PRL 87 230404 (2001)

- By using collective interaction of the atoms with cavity a polarization entangled state of photons could be generated inside a cavity

- Which could be out-coupled and converted to NOON via linear optics.
Bootstrapping

- Generation of NOON states with N roughly 30 with cavity QED based NOON gun.
- Use of Phaseonium to obtain cross-Kerr nonlinearity and the NOON with N=30 as a control in the Quantum Fredkin Gate to generate high NOON states.
- Strong light-atom interaction in cavity QED can also be used to directly implement Quantum Fredkin gate.