QUANTUM SENSORS:
WHAT’S NEW WITH NOON STATES?

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Statue Antiche di Firenze
(Ancient Statues of Florence)

Mother with Children

Scully with Projector
Hearne Institute for Theoretical Physics
Quantum Science & Technologies Group


Not Shown: R.Beaird, M.A. Can, A.Chiruvelli, GA.Durkin, M.Erickson, L. Florescu, M.Florescu, M.Han, KT.Kapale, SJ. Olsen, S.Thanvanthri, Z.Wu, J.Zuo
Outline

1. Quantum Computing & Projective Measurements
2. Quantum Imaging, Metrology, & Sensing
3. Showdown at High NOON!
4. Efficient NOON-State Generating Schemes
5. Conclusions
The objective of the DARPA Quantum Sensor Program is to develop practical sensors operating outside of a controlled laboratory environment that exploit non-classical photon states (e.g. entangled, squeezed, or cat) to surpass classical sensor resolution.
Two Roads to Optical CNOT

I. Enhance Nonlinear Interaction with a Cavity or EIT — Kimble, Walther, Lukin, et al.

II. Exploit Nonlinearity of Measurement — Knill, LaFlamme, Milburn, Franson, et al.
WHY IS A KERR NONLINEARITY LIKE A PROJECTIVE MEASUREMENT?
A Revolution in Nonlinear Optics at the Few Photon Level: No Longer Limited by the Nonlinearities We Find in Nature!

\[ Q = \frac{\pi \hbar}{2} (5\hat{n} - \hat{n}^2) \]

KLM CSIGN Hamiltonian

\[ Q = \frac{\pi \hbar}{2} (3 + a_b^\dagger (1 - \hat{n}_b) + (1 - \hat{n}_b) a_b) \hat{n}_a \]

Franson CNOT Hamiltonian
You want to know if there is a single photon in mode $b$, without destroying it.

Cross-Kerr Hamiltonian: $H_{\text{Kerr}} = \kappa \ a^\dagger a \ b^\dagger b$

Again, with $\kappa = 10^{-22}$, this is impossible.

The success probability is less than 1 (namely 1/8).

The input state is constrained to be a superposition of 0, 1, and 2 photons only.

Conditioned on a detector coincidence in $D_1$ and $D_2$.

Effective $\kappa = 1/8$ → 21 Orders of Magnitude Improvement!

$|\psi_{\text{in}}\rangle = \sum_{n=0}^{2} c_n |n\rangle |1\rangle$

P Kok, H Lee, and JPD, PRA 66 (2003) 063814
Quantum Metrology with NOON States

MACH-ZEHNDER INTERFEROMETER

We call the state of the form $|N, \phi> + |\phi, N>$ the NOON state, and the High NOON state a large $N$.

$\Delta \phi: 1/\sqrt{N} \rightarrow 1/N$

Shotnoise to Heisenberg Limit

Supersensitivity!
FROM QUANTUM INTERFEROMETER TO QUANTUM LITHOGRAPHY

N Photons

\[ |N\rangle_A |0\rangle_B + e^{iN \varphi} |0\rangle_A |N\rangle_B \]

N-Photon Detector

magic BS

\[ |\frac{N}{2}\rangle_A |\frac{N}{2}\rangle_B \]

N-XOR Gates

Mirror

\[ \langle \psi | a^\dagger_N a^N |\psi \rangle \]

Oscillates in REAL Space!

\[ \frac{1 + \cos \varphi}{2} \text{ uncorrelated} \]

\[ \frac{1 + \cos N\varphi}{2} \text{ correlated} \]

\[ \varphi = kx \]

\[ \varphi \rightarrow N\varphi \]

\[ \lambda \rightarrow \lambda/N \]

Superresolution!
How do we make High-NOON!?

\[ |N,0\rangle + |0,N\rangle \]

With a large cross-Kerr nonlinearity!*

\[ H = \kappa \ a^\dagger a \ b^\dagger b \]

This is not practical! — need \( \kappa = \pi \) but \( \kappa = 10^{-22} ! \)

Solution: Replace the Kerr with Projective Measurements!

De Broglie wavelength of a non-local four-photon state

Philip Walther\textsuperscript{1}, Jian-Wei Pan\textsuperscript{1,}\textsuperscript{x}, Markus Aspelmeyer\textsuperscript{1}, Rupert Ursin\textsuperscript{1}, Sara Gasparoni\textsuperscript{1} & Anton Zeilinger\textsuperscript{1,2}

Super-resolving phase measurements with a multiphoton entangled state

M. W. Mitchell, J. S. Lundeen & A. M. Steinberg
A statistical distinguishability based on relative entropy characterizes the fitness of quantum states for phase estimation. This criterion is used to interpolate between two regimes, of local and global phase distinguishability.

The analysis demonstrates that, in a passive MZI, the Heisenberg limit is the true upper limit for local phase sensitivity — \textit{and Only NOON States Reach It!}
NOON–States Violate Bell’s Inequalities


Probabilities of correlated clicks and independent clicks
\[ P_{ab}(\alpha, \beta), P_a(\alpha), P_b(\beta) \]

Building a Clauser–Horne Bell inequality from the expectation values
\[ P_{ab}(\alpha, \beta), P_a(\alpha), P_b(\beta) \]

\[-1 \leq P_{ab}(\alpha, \beta) - P_{ab}(\alpha', \beta') + P_{ab}(\alpha', \beta) + P_{ab}(\alpha', \beta') - P_a(\alpha') - P_b(\beta) \leq 0\]
Efficient Schemes for Generating NOON States!

Question: Do there exist operators “U” that produce “NOON” States Efficiently?

Answer: YES!

H Cable, R Glasser, & JPD, quant-ph/0704.0678. Linear!
N VanMeter, P Lougovski, D Uskov, JPD, quant-ph/0612154. Linear!
KT Kapale & JPD, quant-ph/0612196. (Nonlinear.)
Quantum POOPer Scooper!

H Cable, R Glasser, & JPD, quant-ph/0704.0678.

2-mode squeezing process

\[ \sum_n p_n |n|n \]

OPO

beam splitter

Old Scheme

linear optical processing

\[ |N0\rangle + |ON\rangle \]

\[ \sqrt{2} \]

How to eliminate the “POOP”?

quant-ph/0608170
G. S. Agarwal, K. W. Chan, R. W. Boyd, H. Cable and JPD
Quantum POOPer Scoopers!

H Cable, R Glasser, & JPD, quant-ph/0704.0678.

“Pizza Pie” Phase Shifter

Spinning glass wheel. Each segment a different thickness.
N00N is in Decoherence-Free Subspace!

Feed Forward based circuit

Generates and manipulates special cat states for conversion to N00N states.

First theoretical scheme scalable to many particle experiments!
This counter example disproves the N00N Conjecture: That $N$ Modes Required for N00N.

The upper bound on the resources scales quadratically!

Upper bound theorem:
The maximal size of a N00N state generated in $m$ modes via single photon detection in $m-2$ modes is $O(m^2)$. 
Conclusions

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