Optical Quantum Imaging, Computing, and Metrology: WHAT’S NEW WITH NOON STATES?

Jonathan P. Dowling

Hearne Institute for Theoretical Physics
Louisiana State University
Baton Rouge, Louisiana
quantum.phys.lsu.edu

07 JUNE 2007 DAMOP-07, Calgary
Outline

1. Quantum Computing & Projective Measurements

2. Quantum Imaging, Metrology, & Sensing

3. Showdown at High NOON!

4. Efficient NOON-State Generating Schemes

5. Conclusions
The Controlled-NOT can be implemented using a Kerr medium:

\[ |0\rangle = |H\rangle \text{ Polarization} \]
\[ |1\rangle = |V\rangle \text{ Qubits} \]

\[ R \text{ is a } \pi/2 \text{ polarization rotation, followed by a polarization dependent phase shift } \pi. \]

Unfortunately, the interaction \( \chi^{(3)} \) is extremely weak*:

\[ 10^{-22} \text{ at the single photon level — This is not practical!} \]


II. Exploit Nonlinearity of Measurement — Knill, LaFlamme, Milburn, Franson, et al.
Linear Optics can be Used to Construct CNOT and a Scaleable Quantum Computer:

\[ \alpha |0\rangle + \beta |1\rangle + \gamma |2\rangle \quad \rightarrow \quad \alpha |0\rangle + \beta |1\rangle - \gamma |2\rangle \]

Knill E, Laflamme R, Milburn GJ
NATURE 409 (6816): 46-52 JAN 4 2001

PRL 89 (13): Art. No. 137901 SEP 23 2002
Correlated input-port, matter-wave interferometer: Quantum-noise limits to the atom-laser gyroscope

Jonathan P. Dowling

**FIG. 3.** In a dual Bose condensate, I start with \( P \) particles in each condensate, with a Hilbert product state \( |\psi\rangle_{\text{before}} = |P\rangle|P\rangle \). Particles are allowed to be incident on input ports \( A \) and \( B \) to the beam splitter. The first click of either detector projects the dual condensate into an entangled state \( |\psi\rangle_{\text{after}} = \{ |P - 1\rangle|P\rangle + |P\rangle|P - 1\rangle \}/\sqrt{2} \), which is needed for Heisenberg-limited interferometry, as per Eq. (19), if I take \( N = 2P - 1 \).
WHY IS A KERR NONLINEARITY LIKE A PROJECTIVE MEASUREMENT?
Projective Measurement Yields Effective “Kerr”!

A Revolution in Nonlinear Optics at the Few Photon Level: No Longer Limited by the Nonlinearities We Find in Nature!

\[ Q = \frac{\pi \hbar}{2} (5 \hat{n} - \hat{n}^2) \]

KLM CSIGN Hamiltonian

\[ Q = \frac{\pi \hbar}{2} (3 + a_b^\dagger (1 - \hat{n}_b) + (1 - \hat{n}_b) a_b) \hat{n}_a \]

Franson CNOT Hamiltonian
You want to know if there is a single photon in mode $b$, without destroying it.

Cross-Kerr Hamiltonian: $H_{\text{Kerr}} = \kappa \ a^\dagger a \ b^\dagger b$

Again, with $\kappa = 10^{-22}$, this is impossible.

The success probability is less than 1 (namely 1/8).

The **input state** is constrained to be a superposition of 0, 1, and 2 photons only.

Conditioned on a detector coincidence in $D_1$ and $D_2$.

**Effective $\kappa = 1/8$ → 21 Orders of Magnitude Improvement!**
Outline

1. Quantum Computing & Projective Measurements
2. Quantum Imaging, Metrology, & Sensing
3. Showdown at High NOON!
4. Efficient NOON-State Generating Schemes
5. Conclusions
Quantum Metrology with NOON States

\[ |N\rangle_A |0\rangle_B + e^{iN\phi} |0\rangle_A |N\rangle_B \]

We call the state of the form \(|N, \phi\rangle + |\phi, N\rangle\) the NOON state, and the High NOON state a large \(N\).

Shotnoise to Heisenberg Limit

\[ \varphi = kx \]

\[ \Delta\varphi: \frac{1}{\sqrt{N}} \rightarrow \frac{1}{N} \]

Supersensitivity!
FROM QUANTUM INTERFEROMETRY TO QUANTUM LITHOGRAPHY

N Photons

\[ \left| N \right>_A \left| 0 \right>_B + e^{in \varphi} \left| 0 \right>_A \left| N \right>_B \]

magic BS

N-Photon Detector

Lithographic Resist

\[ \langle \psi | a^\dagger a^N | \psi \rangle \]

\[ \begin{array}{c}
\frac{1 + \cos \varphi}{2} \\
\frac{1 + \cos N \varphi}{2}
\end{array} \]

uncorrelated

correlated

Oscillates in REAL Space!

Superresolution!
Quantum Lithography Experiment

Quantum lithography: setup

- Milena D’Angelo, Maria V. Chekhova, and Yanhua Shih,
PRL 87, 013602 (2001)

Two-photon source: Degenerate Collinear type-II SPDC

|20⟩+|02⟩

Results

|10⟩+|01⟩

Counts per sec

\[ R_c(\theta) = \sin^2[(2\pi a/\lambda) \theta] \cos^2[(2\pi b/\lambda) \theta] \]

\[ I(\theta) = \sin^2[(\pi a/\lambda) \theta] \cos^2[(\pi b/\lambda) \theta] \]
Suppose we have an ensemble of $N$ states $|\varphi\rangle = (|0\rangle + e^{i\varphi} |1\rangle)/\sqrt{2}$, and we measure the following observable: $A = |0\rangle \langle 1| + |1\rangle \langle 0|$

The expectation value is given by: $\langle \varphi|A|\varphi\rangle = N \cos \varphi$
and the variance $(\Delta A)^2$ is given by: $N(1-\cos^2 \varphi)$

The unknown phase can be estimated with accuracy:

$$\Delta \varphi = \frac{\Delta A}{|d \langle A\rangle/d\varphi|} = \frac{1}{\sqrt{N}}$$

This is the standard shot-noise limit.

Now we consider the state \( |\varphi_N\rangle = (|N,0\rangle + |0,N\rangle) \)
and we measure
\[ A_N = |0,N\rangle\langle N,0| + |N,0\rangle\langle 0,N| \]

Quantum Lithography*:
\[ \langle \varphi_N | A_N | \varphi_N \rangle = \cos N\varphi \]

Quantum Metrology:
\[ \Delta \varphi_H = \frac{\Delta A_N}{|d \langle A_N \rangle / d \varphi|} = \frac{1}{N} \]

Heisenberg Limit: No Square Root!

1. Quantum Computing & Projective Measurements

2. Quantum Imaging, Metrology, & Sensing

3. Showdown at High NOON!

4. Efficient NOON-State Generating Schemes

5. Conclusions
Showdown at High-NOON!

How do we make High-NOON!?

\[ |N,0\rangle + |0,N\rangle \]

With a large cross-Kerr nonlinearity!*

\[ H = \kappa \; a^+a \; b^+b \]

This is not practical! — need \( \kappa = \pi \) but \( \kappa = 10^{-22} \)!

Solution: Replace the Kerr with Projective Measurements!

Entangled photons conspire to create interference patterns that would normally be associated with a wavelength much smaller than that of the individual photons — beating the diffraction limit.

It would be more interesting if $|N0,0N\rangle$ states could be generated with $N>2$ but using photons produced by light sources that have a wavelength of at least $\lambda/2$. The existence of such states — dubbed ‘high NOON’ states by Jonathan Dowling — would be an unambiguous demonstration that the diffraction limit has been beaten. This is exactly what Mitchell et al.\textsuperscript{2} and Walther et al.\textsuperscript{3} have achieved, with $|N0,0N\rangle$ states for $N=3$ and $N=4$, respectively.
De Broglie wavelength of a non-local four-photon state

Philip Walther¹, Jian-Wei Pan¹, Markus Aspelmeyer¹, Rupert Ursin¹, Sara Gasparoni¹ & Anton Zeilinger¹,²

Super-resolving phase measurements with a multiphoton entangled state

M. W. Mitchell, J. S. Lundeen & A. M. Steinberg
A statistical distinguishability based on relative entropy characterizes the fitness of quantum states for phase estimation. This criterion is used to interpolate between two regimes, of local and global phase distinguishability.

The analysis demonstrates that, in a passive MZI, the Heisenberg limit is the true upper limit for local phase sensitivity — and Only NOON States Reach It!
NOON–States Violate Bell’s Inequalities


Probabilities of correlated clicks and independent clicks

\[ P_{ab}(\alpha,\beta), P_a(\alpha), P_b(\beta) \]

Building a Clauser–Horne Bell inequality from the expectation values

\[ P_{ab}(\alpha,\beta), P_a(\alpha), P_b(\beta) \]

\[ -1 \leq P_{ab}(\alpha,\beta) - P_{ab}(\alpha,\beta') + P_{ab}(\alpha',\beta) + P_{ab}(\alpha',\beta') - P_a(\alpha') - P_b(\beta) \leq 0 \]

Shared Local Oscillator Acts As Common Reference Frame!
Outline

1. Quantum Computing & Projective Measurements

2. Quantum Imaging, Metrology, & Sensing

3. Showdown at High NOON!

4. Efficient NOON-State Generating Schemes

5. Conclusions
Efficient Schemes for Generating NOON States!

Question: Do there exist operators \( U \) that produce \( \text{"NOON"} \) States Efficiently?

Answer: YES!

H Cable, R Glasser, & JPD, quant-ph/0704.0678. Linear!
N VanMeter, P Lougovski, D Uskov, JPD, quant-ph/0612154. Linear!
KT Kapale & JPD, quant-ph/0612196. (Nonlinear.)
Quantum POOPer Scooper!

H Cable, R Glasser, & JPD, quant-ph/0704.0678.

How to eliminate the “POOP”?

quant-ph/0608170
G. S. Agarwal, K. W. Chan, R. W. Boyd, H. Cable and JPD
Spinning glass wheel. Each segment a different thickness. N00N is in Decoherence-Free Subspace!

Feed-Forward-Based Circuit
Generates and manipulates special cat states for conversion to N00N states.
First theoretical scheme scalable to many particle experiments!
This counter example disproves the N00N Conjecture: That N Modes Required for N00N.

The upper bound on the resources scales quadratically!

Upper bound theorem:

The maximal size of a N00N state generated in \( m \) modes via single photon detection in \( m-2 \) modes is \( O(m^2) \).
Conclusions

1. Quantum Computing & Projective Measurements

2. Quantum Imaging & Metrology

3. Showdown at High NOON!

4. Efficient NOON-State Generating Schemes

5. Conclusions