Consider type I downconversion in degenerate spontaneous parametric downconversion.

\[ \omega_s = \omega_i = \omega = \frac{\omega_p}{2} \]

Recall the photons are bunched.

\[ n_s = \frac{1}{2} \omega_p \rightarrow \text{coherence time of pump} \]

Hence the relative time of arrival at BS:

\[ |1i_i, 1i_i> \text{ has } \Delta z \approx 10-100 \text{ fs accuracy} \]

Let's assume \( L_A = L_B \) initially so \( |1>_{s} \) and \( |1>_{i} \) arrive at BS at same time \( \Delta z = 0 \)

If modes well matched \( \omega_s \approx \omega_i = \omega_p/2 \) and photons are indistinguishable then we can write down BS XFRM.
\[ |1N\rangle = |1_A\rangle |1_B\rangle \]

Recall Eq. 6.17

\[ \hat{a}^+ \rightarrow \frac{1}{\sqrt{2}} \left[ \hat{c}^+ + i \hat{d}^+ \right] \]
\[ \hat{b}^+ \rightarrow \frac{1}{\sqrt{2}} \left[ i \hat{c}^+ + \hat{d}^+ \right] \]

\[ |{\text{out}}\rangle = \hat{a}^+ \hat{b}^+ |0\rangle_A |0\rangle_B \]
\[ = \frac{1}{2} \left[ \hat{c}^+ + i \hat{d}^+ \right] \left[ i \hat{c}^+ + \hat{d}^+ \right] |0\rangle_c |0\rangle_D \]
\[ = \frac{1}{2} \left[ i \hat{c}^+ \hat{c}^+ + \hat{c}^+ \hat{d}^+ - \hat{c}^+ \hat{d}^+ + i \hat{d}^+ \hat{d}^+ \right] |0\rangle_c |0\rangle_D \]
\[ \text{Amplitudes cancel} \]
\[ = \frac{i}{2} \left[ \hat{c}^+ \hat{d}^+ \right] |0\rangle_c |0\rangle_D \]
\[ = \frac{i}{2} \left[ \hat{c}^+ |0\rangle_c |0\rangle_D + \hat{d}^+ |0\rangle_c |0\rangle_D \right] \]
\[ = \frac{i}{2} \left[ \hat{c}^+ |0\rangle_c |2\rangle_D \right] \]

This is a noisy state.
The coincidence counter implements
\[ \hat{D} = \hat{c}^+ \hat{c} \hat{d}^+ \hat{d} \]
That is, \( \hat{D} \) "fires" only when at least one photon in \( C \) and one photon in \( D \) is present. This is a \( g^{(2)} \) type measurement ala Hanbury Brown Twiss.

**IF WE COUNT BEFORE B.S.**

\[
\langle \text{IN} | \hat{D}_{ab} | \text{IN} \rangle = \langle \text{IN} | \hat{a}^+ \hat{c} \hat{a} \hat{b}^+ \hat{b} | \text{IN} \rangle = \langle a | \hat{a}^+ \hat{a} | a \rangle \langle b | \hat{b}^+ \hat{b} | b \rangle = 1
\]

Perfect 100% COINCIDENCE!

**IF WE COUNT AFTER B.S.**

\[
\langle \text{OUT} | \hat{D}_{cd} \hat{D}_{cd} | \text{OUT} \rangle = -\frac{i}{2} [ \langle 21 | c \hat{b}^+ | 21 \rangle + \langle 01 | c \hat{b}^+ | 12 \rangle ] \left[ \hat{c}^+ \hat{c} \hat{d}^+ \hat{d} \right]
\]
\[
= \frac{i}{4} \left[ \langle 21 | \hat{c} \hat{c} | 21 \rangle \langle 21 | \hat{c} \hat{c} | 21 \rangle + \langle 01 | \hat{c} \hat{c} | 12 \rangle \langle 12 | \hat{c} \hat{c} | 21 \rangle + \langle 01 | \hat{c} \hat{c} | 12 \rangle \langle 12 | \hat{c} \hat{c} | 21 \rangle + \langle 01 | \hat{c} \hat{c} | 12 \rangle \langle 12 | \hat{c} \hat{c} | 21 \rangle \right] = 0
\]

ZERO COINCIDENCE! \( \delta \theta = 0 \)
IF $L_A \neq L_B$ THEN PHOTONS BECOME

DISTINGUISHABLE BY ARRIVAL TIME

$\tau_A = L_A/c \neq \tau_B = L_B/c$

AS DISTINGUISHABILITY INCREASES SO DO

COINCIDENCE COUNTS

$I_{\text{Coin}} = 1 - e^{-\Delta\omega^2 [\tau_A - \tau_B]^2}$

$\Delta\omega$ IS LINE WIDTH OF VIRTUAL LEVEL

$\Delta\omega = \frac{1}{T}$ WHERE

$T \approx 10-200 \text{ fs}$

ALLows ONE TO TIME ARRIVAL OF PHOTONS

WITH $\text{fs}$ ACCURACY USING DETECTORS THAT

HAVE ONLY $\text{ns}$ TO $\text{us}$ ACCURACY!

HONG ON MANDDEL DIP

$I_{\text{Coin}}$

100%

10-200 fs

$\Delta 2 \approx 2\tau_A - \tau_B$