The classical theory is in Loudon's Quantum Optics §1.10.
I've put links to both on web page.

A FP interferometer is composed of two partially reflective mirrors.
Each mirror is a reciprocal device and we can use the same 2x2 matrix.
\[ \hat{S} = \hat{N} = \begin{pmatrix} t' & r' \\ r & t \end{pmatrix} \]

The complication is that the field bounces back and forth in the cavity an infinite number of times requiring 2x2 applications of \( \hat{m} \). You can do it this way — summing two geometric series for each mirror but there is a simpler direct method in Loudon's book. Note the BS process becomes

For a single mirror.
For two mirrors we must deal with reflections and cavity field buildup

\[
\begin{align*}
E_0' & \rightarrow E_L' \rightarrow E_2 \leftarrow E_2' \rightarrow E_0'' \\
E_0 & \rightarrow E_L \rightarrow E_2 \leftarrow E_2' \leftarrow 0 = E_1''
\end{align*}
\]

We explicitly assume zero field in from right.
\(E_L\) is cavity field on left mirror.
\(E_R\) is cavity field on right mirror.
Also we may simplify by \(E_0' = 1\) a unit amplitude field which has phase = 0 at L-mirror.
Also to simplify we assume \(\hat{E}_L = \hat{E}_R = (e^{i \phi_2})\)
that is transmission and reflection for two mirrors is the same.

Since \(E_L \neq E_R\) we must find a transfer between them. Ignoring time dependence

\[
E = E e^{i \phi_2} \quad E = E e^{-i \phi_2}
\]

at the left mirror
\[
E_L = r E_{L} e^{2 i \phi_L}
\]

where \(r\) represents reflection of \(R\)-mirror and \(2 i \phi_L\) is round trip phase \(L \Rightarrow R \Rightarrow L\).
Using transfer matrix at L - mirror and R - mirror

\[
\begin{bmatrix}
\begin{bmatrix}
E_r^L \\
E_l^L \\
R_3
\end{bmatrix}
= \begin{bmatrix}
t' & r & \lambda \\
-r' & t & \lambda \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
1_o \\
E_i^L \\
E_i^R
\end{bmatrix}
\end{bmatrix}
\]

\[1_o \rightarrow \begin{bmatrix}
E_r^L \\
E_l^L \\
E_r^R
\end{bmatrix} \rightarrow \begin{bmatrix}
E_l^R \\
E_i^L \\
E_i^R
\end{bmatrix} \leftarrow 0 \cdot \frac{1}{1}
\]

\[E_r^L = t' + r E_i^L \\
E_l^L = r' + t E_i^L
\]

at L where

\[r, t, t', r' \text{ are given}
\]

\[E_l^R = t' E_i^R + 0 \\
E_r^R = r' E_i^R
\]

We have 6 equations and 6 unknowns \( R_3, E_r^L, E_l^L, E_r^R, E_l^R \) and \( E_{out} \). Also we can reduce to 4 unknowns by setting \( E_r^L = \lambda E_i^L = E \) and this is because

(1) takes the round trip phase difference into account,

\[E = \Gamma_3 E \Rightarrow \text{eq} \Rightarrow \phi = 2k \lambda L
\]

\[E = t' + r E \Rightarrow \text{II} \Rightarrow \Gamma_3 = E_{REF}
\]

\[E_{out} = t' E e^{i \varphi} \Rightarrow \text{IV}
\]

\[E = r' E \Rightarrow \text{V}
\]

We now have 4 unknowns \( E, \Gamma_3, E_{out} \) so I - IV are enough to solve everything.
\[ I \rightarrow \Pi \Rightarrow \bar{E} = t' + r \left[ r \bar{E} e^{i \phi} \right] \]

\[ \Rightarrow \bar{E} - r^2 e^{i \phi} \bar{E} = t' \]

\[ \Rightarrow \bar{E} = \frac{t'}{1 - r^2 e^{i \phi}} \]

\[ \Rightarrow \bar{E} = r \left[ \frac{t'}{1 - r^2 e^{i \phi}} \right] e^{i \phi} = \frac{r t' e^{i \phi}}{1 - r^2 e^{i \phi}} \]

As is louder

\[ D = 1 - r^2 e^{i \phi} \]

and

\[ |D|^2 = (1 - r^2 e^{i \phi})(1 - r^2 e^{-i \phi}) \]

\[ = 1 - r^2 e^{i \phi} - r^2 e^{-i \phi} + R^2 \]

\[ = 1 - R e^{i(\phi + 2\psi)} - R e^{-i(\phi + 2\psi)} + R^2 \]

\[ = 1 - 2 R \cos(\phi + 2\psi) + R^2 \]

we may compute intensities

\[ I = |\bar{E}|^2 \text{ in the cavity} \]

\[ I = |\bar{E}|^2 = \frac{RT}{|D|^2} \]

\[ \bar{I} = |\bar{E}|^2 = \frac{RT}{|D|^2} \]

\[ I_{\text{cav}} = \frac{RT + T}{|D|^2} \]

If we take \( \psi = \pi \) the phase on reflection

then \( I_{\text{cav}} \) occurs at \( \cos \phi = 1 \Rightarrow \phi = 2\pi m \)

\[ 2kL = 2\pi m \Rightarrow kL = \pi m \Rightarrow \frac{2\pi L}{\lambda} = \pi m \]

\[ \Rightarrow \left\{ \begin{array}{l} L = \frac{m \lambda}{2} \\ m = 1, 2 \ldots \end{array} \right. \]

Resonance Conditions \( m = 0, 1, 2 \ldots \) Integer # of \( \frac{1}{2} \lambda \) across
Take $\psi = \pi$ and $\varphi = 2k\lambda = \frac{4\pi L}{\lambda} = 2\pi x$

where $\pi = \frac{1}{\sqrt{\lambda}}$ and $R = T = 0.5$

![Graph showing oscillations]

Note $I_{\text{cav}} = 3 > 1$ at max (constructive)

$I_{\text{cav}} = 0.5 < 1$ at min (destructive)

$$I_{\text{cav}}^{\text{max}} = \frac{RT + T}{(1 - R)^2} = \frac{(R + 1)T}{T^2} = \frac{1 + R}{T} = \frac{1.5}{0.5} = 3$$

$$I_{\text{cav}}^{\text{min}} = \frac{RT + T}{(1 + R)^2} = \frac{T(1 + R)}{(1 + R)^2} = \frac{T}{1 + R} = \frac{0.5}{1.5} = 0.33$$

We now compute $I_{\text{REF}} = |\Gamma_3|^2 = |\Gamma_{\text{ref}}|^2$

and $I_{\text{out}} = |\Gamma_{\text{out}}|^2$

$$\Gamma_3 = \Gamma_{\text{ref}} = r' + t \left[ \frac{\Gamma t e^{i\varphi}}{D} \right] = \frac{r'D + rtte^{i\varphi}}{D}$$

Recall $r = \sqrt{R} e^{i\psi}$, $r' = \sqrt{R} e^{i\psi'}$, $t = \sqrt{T} e^{i\theta}$, $t' = \sqrt{T} e^{i\theta'}$

$$\Gamma_{\text{ref}} = \sqrt{R} e^{i\frac{D}{2}} \left[ 1 - \sqrt{R} e^{i\psi'} \right] + \sqrt{R} T e^{i(\psi + \theta + \theta' + \varphi)}$$

$$= e^{i\sqrt{R}} \left\{ D - T e^{i(\psi + \theta + \theta' + \varphi)} \right\}$$

But since (\varphi + \varphi) - (\psi + \psi) = \pi (\text{Reciprocity})

So $\psi - \psi + \theta + \theta = 2\psi + \pi$
\[ E_{\text{REF}} = \frac{e^{i\psi'}}{D} \{ D + T e^{-i(\phi+2\psi')} \} \]

\[ |E_{\text{REF}}|^2 = \frac{R}{|D|^2} \left[ D^* + T \right] \]

\[ = \frac{e^{i\psi'}}{D} \left[ 1 - R e^{i\phi} e^{-i\phi} + T e^{-i(\phi+2\psi')} \right] \]

Again we may take \( \psi = \pi \)

\[ = \frac{e^{i\psi'}}{D} \left[ 1 - R e^{i\phi} + T e^{-i\phi} \right] \]

\[ = \frac{e^{i\psi'}}{D} \left[ 1 + (T-R) e^{-i\phi} \right] \quad T + 1R = 1 \]

\[ = \frac{1}{1 - 2R \cos\phi + R^2} \quad T = 1-R \]

\[ |E_{\text{REF}}|^2 = \frac{R}{|D|^2} \left[ 1 + 2(T-R) \cos\phi + (T-R)^2 \right] \]

\[ = \frac{R \left[ 1 + 2(T-R) \cos\phi +(T-R)^2 \right]}{1 - 2R \cos\phi + R^2} \]

Similarly

\[ E_{\text{OUT}} = \frac{t'e^{i\phi}}{D} = \frac{t' e^{i\phi}}{D} \]

\[ \Rightarrow |E_{\text{OUT}}|^2 = |E_{\text{OUT}}|^2 = \frac{T^2}{|D|^2} = \frac{T^2}{1 - 2R \cos\phi + R^2} \]

Note \(|D|^2 = 1 - 2R \cos\phi + (1-T)^2 \)

\[ = 1 - 2R \cos\phi + 1 - 2T + T^2 \]

\[ = 1 - 2R \cos\phi + 1 - 2(1-R) + T^2 \]

\[ = 2 - 2R \cos\phi - 2 + 2R + T^2 \]

\[ = T^2 + 4R \left( 1 - \cos\phi \right) = \left[ T^2 + 4R \cos^2 \phi / 2 \right] \]
With some work we get \( I_{\text{out}} + I_{\text{ref}} + \frac{d}{d \theta} I_{\text{out}} = 1 \).

In most experiments we are only interested (measure) \( I_{\text{out}} \).

\[
I_{\text{out}} = \frac{T^2}{T^2 + 4R \pi^2 \Phi/2} = \frac{1}{1 + 4R \pi^2 \Phi/2}
\]

Again this has max \( \Rightarrow \Phi/2 = m \pi \Rightarrow \lambda L = m \pi \)

\( \Rightarrow \frac{2\pi L}{\lambda} = m \pi \Rightarrow L = m \lambda/2 \) \( \text{Transmission} \)

resonance condition is same \( L \) where cavity field is max.

\[
I_{\text{out}} = \frac{T^2}{T^2} = 1 \quad \text{Hence Transmission at resonance = 100%}
\]

Let \( \frac{\Phi}{2} = k \cdot L = \frac{2\pi}{\lambda} L = \pi \left( \frac{L}{\lambda/2} \right) = \pi m \) as before

We define \( F = \frac{4R}{T^2} = \frac{4R}{(1-R)^2} \) as the "finesse" of the cavity. Often we just specify \( F \) instead of \( T \) or \( R \) and derive

\( (1-R)^2 F = 4R \Rightarrow (1-2R + R^2)F = 4R \Rightarrow F - 2FR + FR^2 = 4R \)

\( \Rightarrow F = 4R + 2FR - FR^2 = 2(2+F)R - FR^2 \)

\( \Rightarrow FR^2 - 2(2+F)R + F = 0 \)

\( \Rightarrow R^2 - 2(2+F)R + 1 = 0 \)

\( \Rightarrow R = \frac{2(2+F)+1 \pm \sqrt{4(2+F)+4 - 4}}{2} \)

\( \Rightarrow R = \frac{(2+F)+1 \pm \sqrt{4(2+F)+4 - 4}}{2} \)
This allows us to plot \( I_{\text{out}} \) as a function of \( x = \frac{\lambda}{2} \) and \( F \in [0, \infty] \) corresponding to \( R \in [0, 1] \). Hence \( F = 0 \Rightarrow R = 0 \) and \( F = \infty \Rightarrow R = 1 \).

Note when \( I_{\text{out}} \) is minimum at \( \frac{\theta}{2} = \frac{(2m+1)\pi}{2} \)
then \( I_{\text{ref}} \) is max

\[
I_{\text{out}} \mid_{\text{min}} = \frac{T^2}{T^2 + 4R} = \frac{1}{1 + F}
\]

\[
I_{\text{ref}} \mid_{\text{max}} = 1 - I_{\text{out}} \mid_{\text{min}} = 1 - \frac{1}{1 + F}
\]

\[
= \frac{T^2 + 4R - T^2}{T^2 + 4R} = \frac{4R}{1 - 2R + R^2 + 4R} = \frac{4R}{(1 + R)^2}
\]

This occurs when

\[
\frac{\theta}{2} = \frac{(2m+1)\pi}{2} \Rightarrow \frac{\lambda x}{L} = \frac{(2m+1)\pi}{2}
\]

\[
L = (\lambda/2) \left( \frac{2m+1}{2} \right) = \lambda \left( \frac{2m+1}{4} \right)
\]

This is the quarter-wave condition for max reflectance used for reflective coatings. For \( T = R = 0.5 \)

\( F = 8 \Rightarrow I_{\text{out}} \mid_{\text{min}} = \frac{1}{9} \)
Hence by changing $L$ and counting peaks we can measure $L = m \lambda/2$ and given $\lambda$ we know $L$.

This is how a FP is used to measure length. The narrower the peaks the better you can measure $L$. The "free spectral range" $\Delta \lambda$ is the distance between two peaks

$$\Delta \lambda = \lambda_+ - \lambda_-$$

First we compute $\Delta \phi = \phi_+ - \phi_- = (m+1)\pi = k_+ L - k_- L \Rightarrow \Delta k L = \pi$

$$k = \frac{2\pi}{\lambda} \Rightarrow \Delta k = \left| \frac{2\pi}{\lambda^2} \right| \Delta \lambda = \frac{\pi}{L}$$

$$\Rightarrow \Delta \lambda = \frac{\lambda^2}{2L}$$

Free Spectral Range in $\lambda$

In Frequency $\Delta k = \Delta \omega = \frac{\pi}{L} \Rightarrow \Delta \omega = \frac{\pi c}{L}$

This is a range in which we expect to get an accurate measurement for $L$.

Note $t = \frac{2\pi}{\Delta \omega} = \frac{2c}{L}$ is just one round-trip travel time in cavity.

We now want the widths $\Delta \lambda$ and $\Delta \omega$ of these peaks which limits our accuracy $\Delta L$.
Recall \[ \mathcal{I}^{\text{opt}}(\phi) = \frac{1}{1 + F \sin^2 \phi / \lambda} \]

Let \( \delta \phi \) be full width \( \pm \) max: \( \delta \phi = \phi_+ - \phi_- \)

where now \[
\frac{1}{\lambda} = \frac{1}{1 + F \sin^2 \left( \phi / \lambda \right)} \Rightarrow 1 + F \sin^2 \left( \phi / \lambda \right) = \frac{1}{\lambda^2}
\]

\[ F \sin^2 \left( \phi / \lambda \right) = 1 \Rightarrow \sin \left( \phi / \lambda \right) = \pm 1 / \sqrt{F} \]

\[ \phi_{\pm} = \pm 2 \arcsin \left( 1 / \sqrt{F} \right) \Rightarrow \delta \phi = 4 \arcsin \left( 1 / \sqrt{F} \right) \]

Hence \( \delta \phi = 2 \delta k \cdot L = \frac{4 \pi}{\lambda} \delta k \cdot L = 4 \frac{\lambda}{\pi} \arcsin \left( 1 / \sqrt{F} \right) \)

\[ \Rightarrow \frac{\delta k}{\lambda} = \frac{2 \pi}{\lambda} \arcsin \left( 1 / \sqrt{F} \right) \]

and \( \delta \phi = \frac{2 \delta \omega}{\omega} \Rightarrow \delta \omega = \frac{2 \pi}{\lambda} \arcsin \left( 1 / \sqrt{F} \right) \)

If \( \delta \sin = 1 \times \), crudely, set \( \delta \phi \cdot \delta n = 1 \)

\[ \delta n = \sqrt{\delta \phi} \quad \delta n = \frac{1}{\sqrt{\lambda}} \quad \bar{n} = 1 \times 1^2 \]

\[ \Rightarrow \delta \phi = \frac{1}{\sqrt{\lambda}} \Rightarrow \delta k \cdot L = \frac{1}{\pi} \arcsin \left( 1 / \sqrt{F} \right) \]

Exact calculation in ley and Loudon 1987.

Note: assume \( \lambda \) fixed \( \Rightarrow \delta k \cdot 2k = \delta \phi = \)
We define a new quantity — also called the finesse! — \( \mathcal{F} \)

\[
\mathcal{F} = \frac{\Delta \lambda}{\delta \lambda} = \frac{\frac{\lambda^2}{2\kappa}}{\frac{\pi \kappa}{\lambda^2} \frac{1}{\lambda^2 \sin(1/\sqrt{\mathcal{F}})}
\]

\[
= \frac{\pi}{2} \frac{1}{\sin(1/\sqrt{\mathcal{F}})}
\]

The \( \mathcal{F} \) has the same property as \( F \)

\( F \to \infty \Rightarrow \mathcal{F} \to \infty \) and \( \delta \lambda \to 0 \)

narrow peaks — high finesse

For \( 1/\sqrt{\mathcal{F}} \ll 1 \Rightarrow F \gg 1 \)

\[
\mathcal{F} \approx \frac{\pi \sqrt{\mathcal{F}}}{2} = \frac{\pi}{2} \frac{\sqrt{\mathcal{R}}}{(1-R)} \approx \frac{\pi}{2} \frac{1}{\sin(1/\sqrt{\mathcal{F}})}
\]

In a similar fashion we define quality factor \( Q \)

\[
Q = \frac{\omega}{\delta \omega} = \frac{\pi c}{L} \frac{L}{2 \pi c \sin(1/\sqrt{\mathcal{F}})}
\]

\[
= \frac{\pi}{2} \frac{1}{\sin(1/\sqrt{\mathcal{F}})} = \mathcal{F}
\]

Using \( \delta \mathcal{E} \cdot \delta \mathcal{t} \cdot \omega \tau = 1 \)

\[
\Rightarrow \delta \mathcal{t} = \frac{1}{\delta \omega} = \frac{\pi \kappa \lambda}{2 \pi c \sin(1/\sqrt{\mathcal{F}})} = \frac{\pi}{2 \pi c \sin(1/\sqrt{\mathcal{F}})}
\]

\[
\tau = \frac{L}{\pi c} \mathcal{F}
\]

This is the "ringdown" time or photon lifetime in the cavity.

So if \( \mathcal{F} = Q = 300 \) photons makes 100 one-way passes or 50 round trips on average.
\[
\Phi = 2 \left( k + 2 \lambda \right) \Rightarrow \lambda = \frac{k}{2 \lambda} + k
\]

We may model loss via \( k \rightarrow k + \Delta k \) up to some dimensionless constant.

This is equal to that in the vector model.

\[
J_1 L = \frac{2 \lambda Q}{V F} = \frac{2 \lambda}{V \sqrt{L F}}
\]

\[
J_1 L = \frac{2 \lambda}{V F} = \frac{A}{V F} = \frac{A}{2 \pi F F}
\]

Filtered noise limit

Finally, \( \Phi = 2 k L \Rightarrow \Phi = 2 k L \)

\[
A \sin \left( C \frac{1}{\sqrt{F}} \right) = 2 \frac{A}{\sqrt{F}} \sin \left( C \frac{1}{\sqrt{F}} \right)
\]
\[ I_{\text{out}} = \frac{T^2 e^{-2A}}{1 - R e^{-A} e^{-2i\phi} - R e^{-A} e^{2i\phi} + R^2 e^{-2A}} \]
\[ = \frac{T^2 e^{-2A}}{1 - 2R e^{-A} \cos \phi + R^2 e^{-2A}} \]

\[ I_{\text{out}} \approx \frac{T^2 e^{-2A}}{(1 - R e^{-A})^2} = \frac{(1 - R)^2 e^{-2A}}{(1 - R e^{-A})^2} \]

which is no longer 1! If \( A \) is large

\[ I_{\text{out}} \approx \frac{T^2 e^{-2A}}{(1 - R)^2} \frac{e^{-2A}}{e^{2R e^{-A}}} \]

\[ I_{\text{out}} \approx \frac{T^2}{(1 - R)^2} \frac{e^{-2A}}{e^{2R e^{-A}}} \]

\[ R = 0.5 \]

Exponential Loss
Beers' Law

We can model a laser amplifier by taking
\[ -\kappa = +g \quad \text{and} \quad -A = +G \quad \text{gain} \]

\[ I_{\text{out}} \approx \frac{T^2}{(1 - R)^2} \frac{e^{-2A}}{e^{2R e^{-A}}} \]

Exponential Gain

\[ 1.0 \]

\[ 0.2 \]

\[ 1.0G \]
my previous estimates of \( d \Phi \) and \( d L \) were in correct. Here's the corrected version.

Let \( \Phi = Q \Phi \) be the total accumulated phase where \( \Phi = 2 \pi L \) is a round trip but we saw \( Q \) tells us on average how many round trips

\[
Q = \frac{\pi}{2} \frac{1}{\sin(\frac{1}{2}F)} \quad F = \frac{4R}{T^2}
\]

Note for a very high \( Q \) cavity \( R \gg 1 \), and \( \frac{F}{T^2} = \frac{4}{\pi^2} \gg 1 \) implies

\[
Q \approx \frac{\pi}{2} \sqrt{F} \approx \frac{\pi}{4}
\]

Now approximate by differentials

\[
\delta \Phi = Q \delta \Phi = 2Q \Phi \delta L
\]

\[
\delta \Phi \delta n = \frac{1}{n} \Rightarrow \delta \Phi = \frac{1}{\sqrt{n}} \quad \text{where} \quad n \gg 1
\]

\[
\Rightarrow 2Q \Phi \delta L = \frac{1}{\sqrt{n}}
\]

\[
\Rightarrow \frac{2\pi}{T} \frac{2\pi}{\lambda} \delta L = \frac{1}{\sqrt{n}} \Rightarrow \delta L = \frac{1}{4\pi^2} \frac{\lambda}{T} \frac{1}{\sqrt{n}}
\]

This is expression in key and leading up to dimensionless factor on order \( 1 \).

\[
\frac{\delta L}{L} = \frac{1}{4\pi^2} \frac{\lambda}{T} \frac{1}{\sqrt{n}}
\]

So in LIGO

- use big \( L \)
- use small \( n \)
- use big \( F \)
- use \( T \ll 1 \)
Similarly, fixing $l$ we may compute minimum detectible $\lambda$

$$\delta \Phi = \mathcal{Q} \delta \Phi = 2 \mathcal{Q} L \frac{2\pi}{l^2} \delta \lambda$$

$$\delta \Phi = \frac{1}{\sqrt{n}}$$

$$\Rightarrow 2 \mathcal{Q} L \frac{2\pi}{l^2} \delta \lambda = \frac{1}{\sqrt{n}}$$

$$\Rightarrow \frac{\delta \lambda}{\lambda} = \frac{1}{4\pi} \frac{2}{L} \frac{1}{Q} \frac{1}{\sqrt{n}}$$

$$= \frac{1}{4\pi} \frac{2}{L} \frac{1}{Q} \frac{1}{\sqrt{n}} \frac{1}{\sqrt{n}}$$

$$\Rightarrow \frac{\delta \lambda}{\lambda} = \frac{1}{4\pi^2} \frac{2}{L} \frac{1}{Q} \frac{1}{\sqrt{n}}$$

So use small $\lambda$

big $L$ \\
big $\mathcal{Q}$ \\
$T < \lambda$
Oops! For $I_{cav}$ I should have calculated

$$I_{cav} = \left| \vec{E} + \vec{E} \right|^2$$

to correctly account for interference.

$$I_{cav} = \left( \vec{E} + \vec{E} \right) \cdot \left( \vec{E} + \vec{E} \right)$$

$$= \left| \vec{E} \right|^2 + 2 \text{Re} \left( \vec{E} \cdot \vec{E} \right) + \left| \vec{E} \right|^2$$

$$= \frac{1}{T} + 2 \text{Re} \left( \vec{E} \cdot \vec{E} \right) + \frac{1}{T}$$

Recall \( \frac{1}{T} = \frac{T^2}{1D^2} \)

Now \( \vec{E} \cdot \vec{E} = \frac{-1}{T} e^{-i \phi} \)

$$= -\frac{T \sqrt{R} e^{i \phi}}{1D^2} \Rightarrow \text{Re} \left( \vec{E} \cdot \vec{E} \right) = -\frac{T \sqrt{R} \cos \phi}{1D^2}$$

$$\Rightarrow I_{cav} = \frac{RT - T \sqrt{R} \cos \phi + T}{T^2 + 4\pi R \sin^2 \phi / 2}$$

\[
\begin{align*}
I_{cav}^{max} &= \frac{RT + 1}{T^2} = R + 1, \text{ same} \quad \text{at } \phi = \pm \pi \\
I_{cav}^{min} &= \frac{RT + T}{T^2 + 4R} \quad \text{same} \\
I_{cav}^{max} &= I_{cav} \mid \phi = \pm \pi \quad = \frac{RT - T \sqrt{R} + T}{T^2} = \left( \frac{R - \sqrt{R} + 1}{T} \right) \\
I_{cav}^{min} &= I_{cav} \mid \phi = (2m+1)\pi = \left( \frac{RT + T \sqrt{R} + T}{T^2 + 4R} \right)
\end{align*}
\]
The corrected $I_{\text{cav}}$ has minima and maxima at some places, but the values differ.

$R = T = 0.5 \Rightarrow I_{\text{cav}}^{\text{max}} = 1.6 > 1, \quad I_{\text{cav}}^{\text{min}} = 0.5 < 1$

$\tau = \frac{\pi}{2}$