Hint Problem 3.14

Start by proving that if $|r\rangle$ is any eigenvector of the position operator $\hat{q}|r\rangle = r|r\rangle$ then (*):

$$\int_0^\infty dr|-r\rangle\langle r| = \sum_{n=0}^\infty (-1)^n |n\rangle\langle n|$$

where the n are the number basis.

Then using the 1D displacement operator $e^{\pm i\xi\hat{p}}|r\rangle = |r \pm \xi\rangle$ you can show:

$$\hat{D}(\alpha)|-r\rangle = e^{-i\frac{pq}{2}} e^{ip(q-r)}|q-r\rangle$$

and

$$\langle r|\hat{D}^\dagger(\alpha) = e^{i\frac{pq}{2}} e^{-ip(q+r)}\langle q+r|$$

Now take the Wigner function definition in *p, q, x,* Eq. 3.116 and set *r = x/2.* Take $\hbar = 1$ and massage until you get:

$$W(\alpha) = \frac{1}{\pi}\int_{-\infty}^{\infty} dr \langle r|\hat{D}^\dagger(\alpha)\hat{\rho}\hat{D}(\alpha)|-r\rangle$$

With the identity (*) above and the fact that $\hat{\rho} = |\Psi\rangle\langle\Psi|$ for a pure state you can get the answer in the book.