Physics 2102
Lecture 05: TUE 02 FEB

Gauss’ Law II

Flux Capacitor (Operational)

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Carl Friedrich Gauss
1777 – 1855
What-Dat™ Flux!

Constant Area:

\[ \Phi_{\text{Flux}} = \vec{E} \cdot \vec{A} = EA \cos \theta \]

Changing Area:

\[ \Phi_{\text{Flux}} = \int \vec{E} \cdot d\vec{A} = \int E \cos \theta \, dA \]
Gauss’ Law: General Case

- Consider any ARBITRARY CLOSED surface S -- NOTE: this “Gaussian Surface” does NOT have to be a “real” physical object!

- The TOTAL ELECTRIC FLUX through S is proportional to the TOTAL CHARGE ENCLOSED!

- The results of a complicated integral is a very simple formula: it avoids long calculations!

\[ \Phi = \oint _{\text{Surface}} \vec{E} \cdot d\vec{A} = \pm \frac{q}{\varepsilon_0} \]  

(One of Maxwell’s 4 equations!)
What is Flux Through Surfaces:

\[ S_1 = +\frac{q}{\varepsilon_0} \]
\[ S_2 = -\frac{q}{\varepsilon_0} \]
\[ S_3 = 0 \]
\[ S_4 = 0 \]
Faraday’s Cage: Electric Field Inside Hollow Conductor is Zero

- Choose any arbitrary surface inside the metal
- Since $E = 0$, flux = 0
- Hence total charge enclosed = 0
  All charge goes on outer surface!

Inside cavity is “shielded” from all external electric fields! “Faraday Cage effect”
Bill Downs takes a look inside the Cage of Death
Gauss’ Law: Cylindrical Symmetry

- Charge of \( q = 10 \text{ C} \) is uniformly spread over a line of length \( L = 1 \text{ m} \).

- Use Gauss’ Law to compute magnitude of \( E \) at a perpendicular distance of 1 mm from the center of the line.

- Approximate as infinitely long line — \( E \) radiates outwards.

- Choose cylindrical surface of radius \( R \), length \( L \) co-axial with line of charge.

Line of Charge:
\[ \lambda = \frac{q}{L} \]
Units: \([\text{C/m}]\)
Gauss’ Law: Cylindrical Symmetry

- Approximate as infinitely long line — E radiates outwards.
- Choose cylindrical surface of radius \( R \), length \( L \) coaxial with line of charge.

\[
\Phi = |E| \cdot A = |E| \cdot 2\pi RL
\]

\[
\Phi = \frac{q}{\varepsilon_0} = \frac{\lambda L}{\varepsilon_0}
\]

\[
|E| = \frac{\lambda L}{2\pi\varepsilon_0 RL} = \frac{\lambda}{2\pi\varepsilon_0 R} = 2k \frac{\lambda}{R}
\]

\( R = 1 \text{ mm} \)

\( \vec{E} = ? \)
Recall Finite Line Example!

\[ E_y = k \lambda a \int_{-L/2}^{L/2} \frac{dx}{(a^2 + x^2)^{3/2}} = k \lambda a \left[ \frac{x}{a^2 \sqrt{x^2 + a^2}} \right]_{-L/2}^{L/2} = \frac{2k\lambda L}{a \sqrt{4a^2 + L^2}} \]

If the Line Is Infinitely Long \((L \gg a)\)

\[ E_y = \frac{2k\lambda L}{a \sqrt{L^2}} = \frac{2k\lambda}{a} \]

\[ a = R \]
Gauss' Law: Insulating Plate

- Infinite INSULATING plane with uniform charge density \( s \)
- \( E \) is NORMAL to plane
- Construct Gaussian box as shown

Applying Gauss' law \( \frac{q}{\varepsilon_0} = \Phi \), we have, \( \frac{A\sigma}{\varepsilon_0} = 2AE \)

Solving for the electric field, we get \( E = \frac{\sigma}{2\varepsilon_0} \)

Surface Charge; \( \sigma = \frac{q}{A} \)
Units: \([C/m^2]\)

For an insulator, \( E=\sigma/2\varepsilon_0 \), and for a conductor, \( E=\sigma/\varepsilon_0 \).
Insulating and Conducting Planes

Insulating Plate: Charge Distributed Homogeneously.

Conducting Plate: Charge Distributed on the Outer Surfaces. Electric Field Inside a Conductor is ZERO!
Gauss’ Law: Spherical Symmetry

- Consider a POINT charge $q$ & pretend that you don’t know Coulomb’s Law
- Use Gauss’ Law to compute the electric field at a distance $r$ from the charge
- Use symmetry:
  - place spherical surface of radius $R$ centered around the charge $q$
  - $E$ has same magnitude anywhere on surface
  - $E$ normal to surface

\[
\Phi = \frac{q}{\varepsilon_0} \quad \quad \quad |E| = \frac{q}{4\pi\varepsilon_0 r^2} = \frac{kq}{r^2}
\]
A spherical shell has a charge of +10C and a point charge of -15C at the center. What is the electric field produced OUTSIDE the shell?

If the shell is conducting?
Field Inside a Conductor is ZERO!

And if the shell is insulating?
Charged Shells Behave Like a Point Charge of Total Charge “Q” at the Center Once Outside the Last Shell!
Electric Fields With Insulating Sphere

**Inside Sphere (r < R):**

\[ q_{\text{enclosed}} = Q \left( \frac{V_{\text{enclosed}}}{V_{\text{total}}} \right) = Q \left( \frac{4\pi r^3 / 3}{4\pi R^3 / 3} \right) = Q \frac{r^3}{R^3} \]

**Outside Sphere (r > R):**

\[ q_{\text{enclosed}} = Q \]

**Potential (\( \Phi \))**:

\[ \Phi = EA = \frac{q_{\text{enclosed}}}{\varepsilon_0} \]

**Field inside sphere (r < R):**

\[ E = \frac{1}{4\pi \varepsilon_0} \frac{Q}{R^2} \]

**Field outside sphere (r > R):**

\[ E = \frac{1}{4\pi \varepsilon_0} \frac{Qr}{r^2} \]

\[ r < R \rightarrow E 4\pi r^2 = Q \frac{r^3}{R^3} / \varepsilon_0 \]

\[ r > R \rightarrow E 4\pi r^2 = Q / \varepsilon_0 \]
Summary

• **Electric Flux**: a Surface Integral (Vector Calculus!); Useful Visualization: Electric Flux Lines Like Wind Through a Window.

• Gauss’ Law Provides a Very Direct Way to Compute the Electric Flux.
Tell us, in layman's terms, what your breakthrough means.

Certainly, $K - \frac{4n^3 \sqrt{P}}{7} + 4 \cdot \frac{E \cdot L}{57}$.