Lecture 41: WED 29 APR
Ch. 36: Diffraction

Physics 2102
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PHYS 2102-2 FINAL

• 5:30-7:30PM FRI 08 MAY
• COATES 143
• 1/2 ON NEW MATERIAL
• 1/2 ON OLD MATERIAL
• Old Formula Sheet: http://www.phys.lsu.edu/classes/spring2007/phys2102/formulasheet.pdf
Radar: The Smaller The Wavelength the Better The Targeting Resolution

- **X-band:** $\lambda = 10\text{cm}$
- **K-band:** $\lambda = 2\text{cm}$
- **Ka-band:** $\lambda = 1\text{cm}$
- **Laser:** $\lambda = 1\ \mu\text{m}$

Radar: Outgoing waves

Radar gun

Reflected waves (higher frequency)
Angles of the Secondary Maxima

The diffraction minima are precisely at the angles where \( \sin \theta = p \lambda/a \) and \( \alpha = p\pi \) (so that \( \sin \alpha = 0 \)).

However, the diffraction maxima are not quite at the angles where \( \sin \theta = (p+\frac{1}{2}) \lambda/a \) and \( \alpha = (p+\frac{1}{2})\pi \) (so that \( |\sin \alpha| = 1 \)).

To find the maxima, one must look near \( \sin \theta = (p+\frac{1}{2}) \lambda/a \), for places where the slope of the diffraction pattern goes to zero, i.e., where \( d((\sin \alpha/\alpha)^2)/d\theta = 0 \). This is a transcendental equation that must be solved numerically. The table gives the \( \theta_{\text{Max}} \) solutions. Note that \( \theta_{\text{Max}} < (p+\frac{1}{2})\lambda/a \).
Example: Diffraction of a laser through a slit

Light from a helium-neon laser (\( \lambda = 633 \) nm) passes through a narrow slit and is seen on a screen 2.0 m behind the slit. The first minimum of the diffraction pattern is observed to be located 1.2 cm from the central maximum.

How wide is the slit?

\[
\theta_1 = \frac{y_1}{L} = \frac{0.012 \text{ m}}{2.00 \text{ m}} = 0.0060 \text{ rad}
\]

\[
a = \frac{\lambda}{\sin \theta_1} \approx \frac{\lambda}{\theta_1} = \frac{(6.33 \times 10^{-7} \text{ m})}{(6.00 \times 10^{-3} \text{ rad})} = 1.06 \times 10^{-4} \text{ m} = 0.106 \text{ mm}
\]
Width of a Single-Slit Diffraction Pattern

\[ y_p = \frac{p\lambda L}{a}; \quad p = 1, 2, 3, \cdots \quad \text{(positions of dark fringes)} \]

\[ w = \frac{2\lambda L}{a} \quad \text{(width of diffraction peak from min to min)} \]
You are doing 137 mph on I-10 and you pass a little old lady doing 55 mph when a cop, located 1 km away fires his radar gun, which has a 10 cm opening. Can he tell you from the L.O.L. if the gun is X-band? What about Laser?

X-band: $\lambda = 10\text{ cm}$

$$w = \frac{2\lambda L}{a} = \frac{2 \times 0.1\text{ m} \times 1000\text{ m}}{0.1\text{ m}} = 2000\text{ m}$$

No! Beam is much wider than car separation — too wide to tell who is who.

Laser-band: $\lambda = 1\mu\text{ m}$

$$w = \frac{2\lambda L}{a} = \frac{2 \times 0.000001\text{ m} \times 1000\text{ m}}{0.1\text{ m}} = 0.02\text{ m}$$

Yes! Beam width is much less than car separation.
Exercise

Two single slit diffraction patterns are shown. The distance from the slit to the screen is the same in both cases. Which of the following could be true?

(a) The slit width $a$ is the same for both; $\lambda_1 > \lambda_2$.
(b) The slit width $a$ is the same for both; $\lambda_1 < \lambda_2$.
(c) The wavelength is the same for both; width $a_1 < a_2$.
(d) The slit width and wavelength is the same for both; $p_1 < p_2$.
(e) The slit width and wavelength is the same for both; $p_1 > p_2$. 
Combined Diffraction and Interference

So far, we have treated diffraction and interference independently. However, in a two-slit system both phenomena should be present together.

\[ I_{\text{2slit}} = 4I_{\text{1slit}} \cos^2 (\delta) \left( \frac{\sin(\alpha)}{\alpha} \right)^2; \]
\[ \alpha = \frac{\pi a}{\lambda L} \quad y = \frac{\pi a}{\lambda} \sin \theta; \]
\[ \delta = \frac{\pi d}{\lambda L} \quad y = \frac{\pi d}{\lambda} \sin \theta. \]

Notice that when \( d/a \) is an integer, diffraction minima will fall on top of “missing” interference maxima.
A device with $N$ slits (rulings) can be used to manipulate light, such as separate different wavelengths of light that are contained in a single beam. How does a diffraction grating affect monochromatic light?

\[ d \sin \theta = m\lambda \quad \text{for} \quad m = 0, 1, 2 \ldots \quad \text{(maxima-lines)} \]

(36-11)
Circular Apertures

If light travels in straight lines, the image on the screen is the same size as the hole. Diffraction will not be noticed unless the light spreads over a diameter larger than $D$.

When light passes through a **circular** aperture instead of a vertical slit, the diffraction pattern is modified by the 2D geometry. The minima occur at about $1.22\lambda/D$ instead of $\lambda/a$. Otherwise the behavior is the same, including the spread of the diffraction pattern with decreasing aperture.
The Rayleigh Criterion says that the minimum separation to separate two objects is to have the diffraction peak of one at the diffraction minimum of the other, i.e., $\Delta \theta = \frac{1.22 \lambda}{D}$.

**Example:** The Hubble Space Telescope has a mirror diameter of 4 m, leading to excellent resolution of close-lying objects. For light with wavelength of 500 nm, the angular resolution of the Hubble is $\Delta \theta = 1.53 \times 10^{-7}$ radians.
Example

A spy satellite in a 200km low-Earth orbit is imaging the Earth in the visible wavelength of 500nm.

How big a diameter telescope does it need to read a newspaper over your shoulder from Outer Space?
Example Solution

\[ \Delta \theta = \frac{1.22 \lambda}{D} \] (The smaller the wavelength or the bigger the telescope opening — the better the angular resolution.)

Letters on a newspaper are about \( \Delta x = 10\text{mm} \) apart. Orbit altitude \( R = 200\text{km} \) & \( D \) is telescope diameter.

Formula:

\[ \Delta x = R \Delta \theta = R \left( \frac{1.22 \lambda}{D} \right) \]

\[ D = R \left( \frac{1.22 \lambda}{\Delta x} \right) \]

\[ = (200 \times 10^3 \text{m}) \left( \frac{1.22 \times 500 \times 10^{-9} \text{m}}{10 \times 10^{-3} \text{m}} \right) \]

\[ = 12.2 \text{m} \]
Los Angeles from Space!
Corona Declassified Spy Photo:
Circa 1960's
Review of Interference from Thin Films

\[ \phi_{12} = ? \]

\[ \theta \approx 0 \]

Fig. 35-15

Soap Bubble

Oil Slick
Reflection Phase Shifts

\[ \lambda = 360^\circ = 2\pi \]

\[ \frac{\lambda}{2} = 180^\circ = \pi \]

Reflection Phase Shift:

Off lower index ---> 0
Off higher index ---> \( \frac{\lambda}{2} = 180^\circ = \pi \)

High-To-Low: Phase Shift — NO!

Low-To-High: Phase Shift — \( \pi \)!
Equations for Thin-Film Interference

Three effects can contribute to the phase difference between $r_1$ and $r_2$.

1. Differences in reflection conditions.

2. Difference in path length traveled.

3. Differences in the media in which the waves travel. One must use the wavelength in each medium ($\lambda/n$) to calculate the phase.

$\frac{1}{2}$ wavelength phase difference to difference in reflection of $r_1$ and $r_2$

\[
2L = \frac{\text{odd number}}{2} \times \text{wavelength} = \frac{\text{odd number}}{2} \times \lambda_{n_2} \quad \text{(in-phase waves)}
\]

\[
2L = \text{integer} \times \text{wavelength} = \text{integer} \times \lambda_{n_2} \quad \text{(out-of-phase waves)}
\]

\[
\lambda_{n_2} = \frac{\lambda}{n_2}
\]

\[
2L = (m + \frac{1}{2}) \frac{\lambda}{n_2} \quad \text{for } m = 0, 1, 2, \ldots \quad \text{(maxima-- bright film in air)}
\]

\[
2L = m \frac{\lambda}{n_2} \quad \text{for } m = 0, 1, 2, \ldots \quad \text{(minima-- dark film in air)}
\]

(35-16)
Film Thickness Much Less Than $\lambda$

If $L$ is much less than $l$, for example $L < 0.1\lambda$, then phase difference due to the path difference $2L$ can be neglected.

Phase difference between $r_1$ and $r_2$ will always be $\frac{1}{2}$ wavelength $\rightarrow$ destructive interference $\rightarrow$ film will appear dark when viewed from illuminated side.

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Top of Soap Bubble
Film is So Thin
It Looks Dark
At All Wavelengths:
Anti-Reflective Coating!

As You Go Down
Thickness Increases Steadily (Gravity)
And Different Wavelengths hit Reflection Condition
$2L=(m+1/2)\lambda/n_2$
Reflective Coating

(35-17)
For the same path difference, different wavelengths (colors) of light will interfere differently. For example, $2L$ could be an integer number of wavelengths for red light but half-integer wavelengths for blue.

Furthermore, the path difference $2L$ will change when light strikes the surface at different angles, again changing the interference condition for the different wavelengths of light.
Problem Solving Tactic 1: Thin-Film Equations

Equations 35-36 and 35-37 are for the special case of a higher index film flanked by air on both sides. For multilayer systems, this is not always the case and so these equations are not appropriate.

What happens to these equations for the following system?

\[
\begin{align*}
n_1 &= 1 \\
n_2 &= 1.5 \\
n_3 &= 1.7
\end{align*}
\]