Lecture 15: MON 16 FEB

DC Circuits

Ch27.1–4

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The battery operates as a “pump” that moves positive charges from lower to higher electric potential. A battery is an example of an “electromotive force” (EMF) device.

These come in various kinds, and all transform one source of energy into electrical energy. A battery uses chemical energy, a generator mechanical energy, a solar cell energy from light, etc.

The difference in potential energy that the device establishes is called the EMF and denoted by $\mathcal{E}$.

$$\mathcal{E} = iR$$
Given the EMF devices and resistors in a circuit, we want to calculate the circulating currents. Circuit solving consists in “taking a walk” along the wires. As one “walks” through the circuit (in any direction) one needs to follow two rules:

When walking through an EMF, add $+\mathcal{E}$ if you flow with the current or $-\mathcal{E}$ against. How to remember: current “gains” potential in a battery.

When walking through a resistor, add $-iR$, if flowing with the current or $+iR$ against. How to remember: resistors are passive, current flows “potential down”.

**Example:**
Walking clockwise from $a$: $+\mathcal{E}-iR=0$.
Walking counter-clockwise from $a$: $-\mathcal{E}+iR=0$. 
Ideal vs. Real Batteries

If one connects resistors of lower and lower value of $R$ to get higher and higher currents, eventually a real battery fails to establish the potential difference $\mathcal{E}$, and settles for a lower value.

One can represent a “real EMF device” as an ideal one attached to a resistor, called “internal resistance” of the EMF device:

$$\mathcal{E}_{\text{true}} = \mathcal{E} - ir$$

The true EMF is a function of current: the more current we want, the smaller $\mathcal{E}_{\text{true}}$ we get.

$$\mathcal{E} - ir - iR = 0 \rightarrow i = \mathcal{E}/(r + R)$$
Resistances in Series: i is Constant

Two resistors are “in series” if they are connected such that the **same current i** flows in both.
The “equivalent resistance” is a single imaginary resistor that can replace the resistances in series.

“Walking the loop” results in:

\[ \mathcal{E} - iR_1 - iR_2 - iR_3 = 0 \rightarrow i = \frac{\mathcal{E}}{(R_1 + R_2 + R_3)} \]

In the circuit with the equivalent resistance,

\[ \mathcal{E} - iR_{eq} = 0 \rightarrow i = \frac{\mathcal{E}}{R_{eq}} \]

Thus,

\[ R_{eq} = \sum_{j=1}^{n} R_j \]
Resistors in Parallel: V is Constant

Two resistors are “in parallel” if they are connected such that there is the **same potential** V drop through both. The “equivalent resistance” is a single imaginary resistor that can replace the resistances in parallel.

“Walking the loops” results in:

\[ \mathcal{E} - i_1 R_1 = 0, \quad \mathcal{E} - i_2 R_2 = 0, \quad \mathcal{E} - i_3 R_3 = 0. \]

The total current delivered by the battery is:

\[
i = i_1 + i_2 + i_3 = \frac{\mathcal{E}}{R_1} + \frac{\mathcal{E}}{R_2} + \frac{\mathcal{E}}{R_3} = \mathcal{E} \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)\]

In the circuit with the equivalent resistor, \[i = \frac{\mathcal{E}}{R_{eq}}\]

\[
\frac{1}{R_{eq}} = \sum_{j=1}^{n} \frac{1}{R_j}
\]
**Resistors**

\[ V = iR \]

**Series: \( I = \frac{dQ}{dt} \) Same**

\[ R_{\text{ser}} = R_1 + R_2 + R_3 + \ldots \]

**Parallel: \( V \) Same**

\[ \frac{1}{R_{\text{par}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots \]

**Capacitors**

\[ Q = CV \]

**Series: \( Q \) Same**

\[ \frac{1}{C_{\text{ser}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots \]

**Parallel: \( V \) Same**

\[ C_{\text{par}} = C_1 + C_2 + C_3 + \ldots \]
Resistors in Series and Parallel

An electrical cable consists of 100 strands of fine wire, each having \( r = 2 \, \Omega \) resistance. The same potential difference is applied between the ends of all the strands and results in a total current of \( I = 5 \, A \).

(a) What is the current in each strand?
Ans: \( i_p = 0.05 \, A \) (\( i = I / 100 \))

(b) What is the applied potential difference?
Ans: \( v_p = 0.1 \, V \) (\( v_p = V = i_s r = \text{constant} \))

(c) What is the resistance of the cable?
Ans: \( R_p = r = 0.02 \, \Omega \) (\( 1 / R_p = 1 / r + 1 / r + \ldots = 100 / r \Rightarrow R = r / 100 \))

Assume now that the same 2 \( \Omega \) strands in the cable are tied in series, one after the other, and the 100 times longer cable connected to the same \( V = 0.1 \) Volts potential difference as before.

(d) What is the potential difference through each strand?
Ans: \( v_s = 0.001 \, V \) (\( v_s = V / 100 \))

(e) What is the current in each strand?
Ans: \( i_s = 0.0005 \, A \) (\( i_s = v_s / r = \text{constant} \))

(f) What is the resistance of the cable?
Ans: 200 \( \Omega \) (\( R_s = r + r + r + \ldots = 100r \))

(g) Which cable gets hotter, the one with strands in parallel or the one with strands in series?

Ans: Each strand in parallel dissipates \( P_p = iv_p = 5 \, mW \) (and the cable dissipates 100 \( \bullet P_p = 500 \, mW \)); Each strand in series dissipates \( P_s = i_s \cdot v_s = 50 \, \mu W \) (and the cable dissipates 5 \( \mu W \))
Example

38E. A circuit containing five resistors connected to a battery with a 12.0 V emf is shown in Fig. 28-38. What is the potential difference across the 5.0 Ω resistor?

Bottom loop: (all else is irrelevant)
V same in parallel!

Which resistor (3 or 5) gets hotter? $P = i^2 R$
a) Which circuit has the largest equivalent resistance?
b) Assuming that all resistors are the same, which one dissipates more power?
c) Which resistor has the smallest potential difference across it?
Example

Find the equivalent resistance between points
(a) $F$ and $H$ and
(b) $F$ and $G$.

(Hint: For each pair of points, imagine that a battery is connected across the pair.)
If all resistors have a resistance of $4\Omega$, and all batteries are ideal and have an emf of $4V$, what is the current through $R$?

If all capacitors have a capacitance of $6\mu F$, and all batteries are ideal and have an emf of $10V$, what is the charge on capacitor $C$?